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Note 04-3: Use of Butterworth filter for lowpass filtering of water level data in Norwegian Hydrographic Service

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1 Introduction

The Norwegian Hydrographic Service is using two Butterworth filters for filtering of water level data. Both filters are lowpass (LP) filters, which operate on data with 10-minutes sampling period, but they have different cut-off frequency[1].

Both filters are 4th order Butterworth filter implemented as a cascade of two 2nd order filters to minimise round of error, which can be a problem with higher order filters [1]. To avoid phase shift in the filtered time series, the original time series is first filtered forward, and then backward in time thorough the same filter. The resultant amplitude response will then be the square of the amplitude response of the filter.

The first filter is an anti-aliasing filter, which is used before decimating from 10-minute values to 1-hour values (before performing harmonic analysis).

The second filter is used to smooth 10 minute values collected after renewing the Norwegian tide gauge network in 2000-2002.

The purpose of this note is to document the two filters.

In NHSs water level database all modern data are stored as 10-minute values. For more information of filtering of the data before they are stored in the database see [8].

2 Filter 1: Anti-aliasing filter

The purpose of this filter is to LP-filter the 10-minute values before decimating to hourly-values, to avoid aliasing.

The filter has been used in NHS since the 1980-ies. A verification of the filter is described in [3].

The amplitude response of the 4th order Butterworth filter is shown in Figure 2-1. The amplitude responses of the two 2nd order filters are shown as well, but it is the resultant response that is important. The cut off frequency is 1/(3 hours) = 0.00009259 Hz.

To avoid aliasing all frequencies above the Nyquist frequency (f_{Nyq}) should be removed before decimating to hourly-values. The Nyquist frequency f_{Nyq} is half of the sampling frequency f_s = 1/(1time) = 0.0002778 Hz.

$$f_{Nyq} = \frac{f_s}{2} = \frac{1}{1time \cdot 2} = \frac{1}{60 \cdot 60 \cdot 2} Hz = 0.0001388...Hz$$

The amplitude response is ca. –14.7 dB[2] (= 0.184) at f_{Nyq} (possible to see when zooming into Figure 2-1). Since the time series is run forward and backward the resultant amplitude response is –29.4 dB (=0.034) at f_{Nyq}. Which means that for all practical purposes this is an acceptable anti-aliasing filter because almost all energy above the Nyquist frequency is filtered away. The squared amplitude response of the 4th order Butterworth filter is shown in Figure 2-2. The squared amplitude response will be the truth amplitude response, since the filter is used twice.

[1] The cut off frequency of a LP-filter is the frequency where the amplitude response is (1/√2) times its value at zero frequency, or equivalently, the frequency at which the gain drops by 3 dB below its value at zero frequency.
[2] -14.7 dB = 20log(0.184)

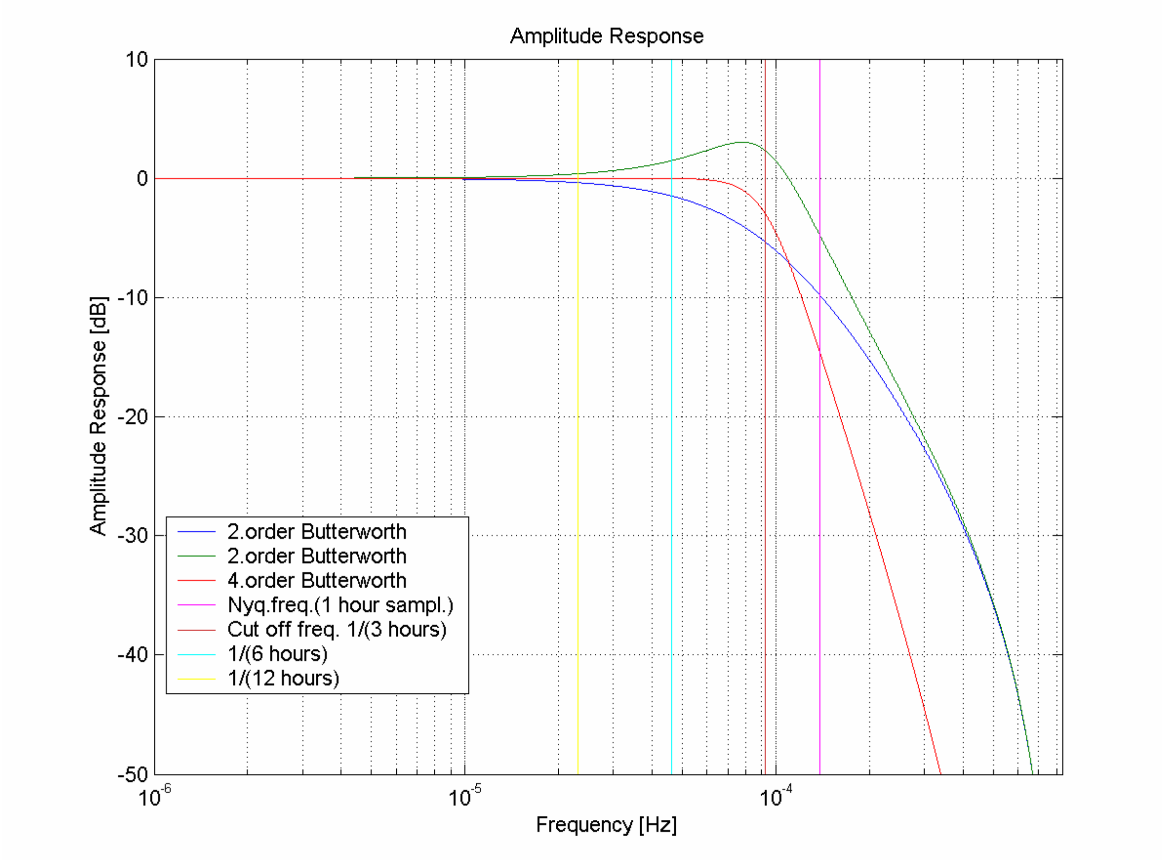

Figure 2-1 Amplitude response (in dB) for a 4th order Butterworth LP filter consisting of 2 cascaded 2nd order filters. Four different frequencies is shown as well: The Nyquist frequency for hourly sampling interval 1/(2 hours), the (-3 dB) cut off frequency 1/(3 hours) for the 4th order filter, and two of the tide frequencies 1/(6 hours) (4th diurnal constituents) and 1/(12 hours) (semidiurnal constituents). Note that the cut off frequency is a tide frequency as well (8th diurnal constituents).

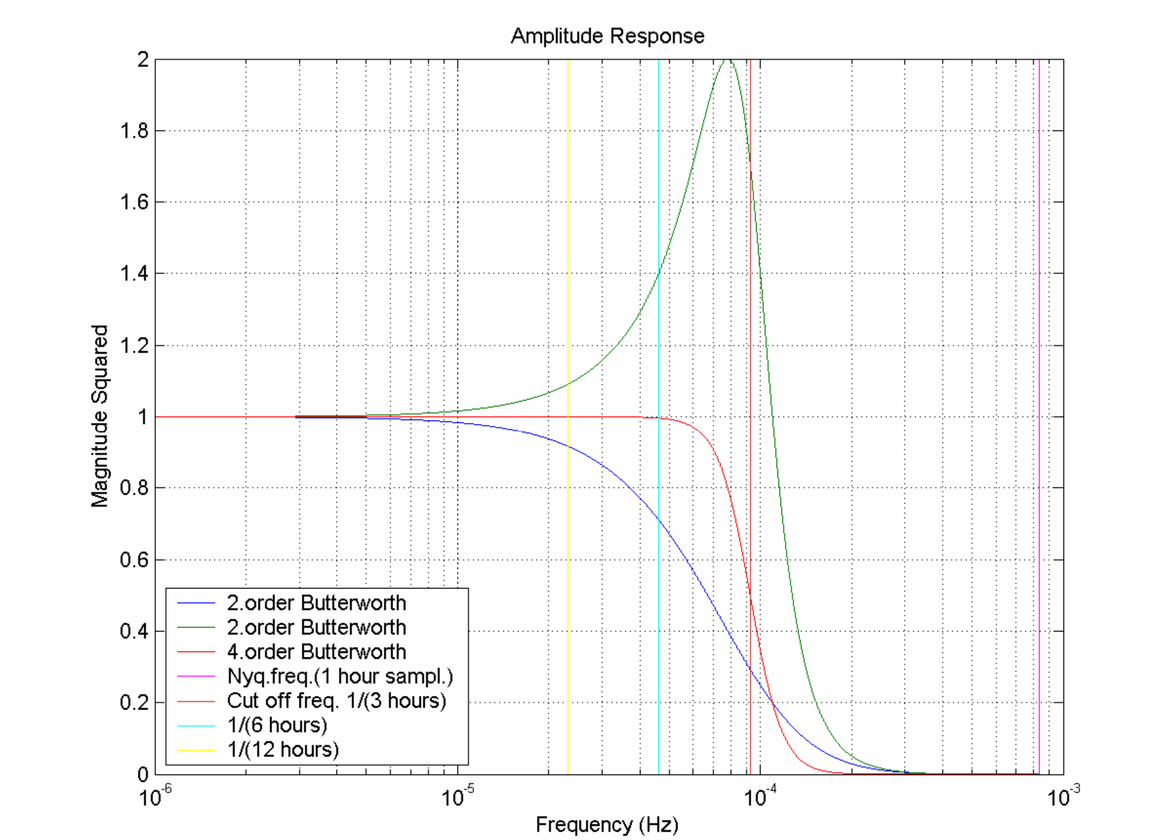

Figure 2-2 Squared amplitude response for a 4th order Butterworth LP filter consisting of 2 cascaded 2nd order filters. Four different frequencies is shown as well: The Nyquist frequency for hourly sampling interval 1/(2 hours), the (-3 dB) cut off frequency 1/(3 hours) for the 4th order filter, and two of the tide frequencies 1/(6 hours) (4th diurnal constituents) and 1/(12 hours) (semidiurnal constituents). Note that the cut off frequency is a tide frequency as well (8th diurnal constituents).

It is important that an anti-aliasing filter removes energy above the Nyquist frequency, but at the same time the tide frequencies and their over-harmonics should be as undisturbed as possible. This is especially important for the data used for *harmonic analysis*. In harmonic analysis we find the harmonic tidal *constants* for the tidal *constituents*. If some of the tidal frequencies are filtered with a filter with an amplitude response different from 1 (= 0 dB) this will affect the amplitudes of the respective constituents. In Figure 2-2 we can see that for the 6-hour and 12-hour tide interval, the squared amplitude response is almost 1. But for the 3-hour tide interval the squared amplitude response is 0.5, which means that only half of the amplitude comes through the filter process. In Figure 2-3 the amplitude response of the filter is shown together with more tide frequencies. We can see that frequencies from the fifth-diurnal and higher will be attenuated by the filter.

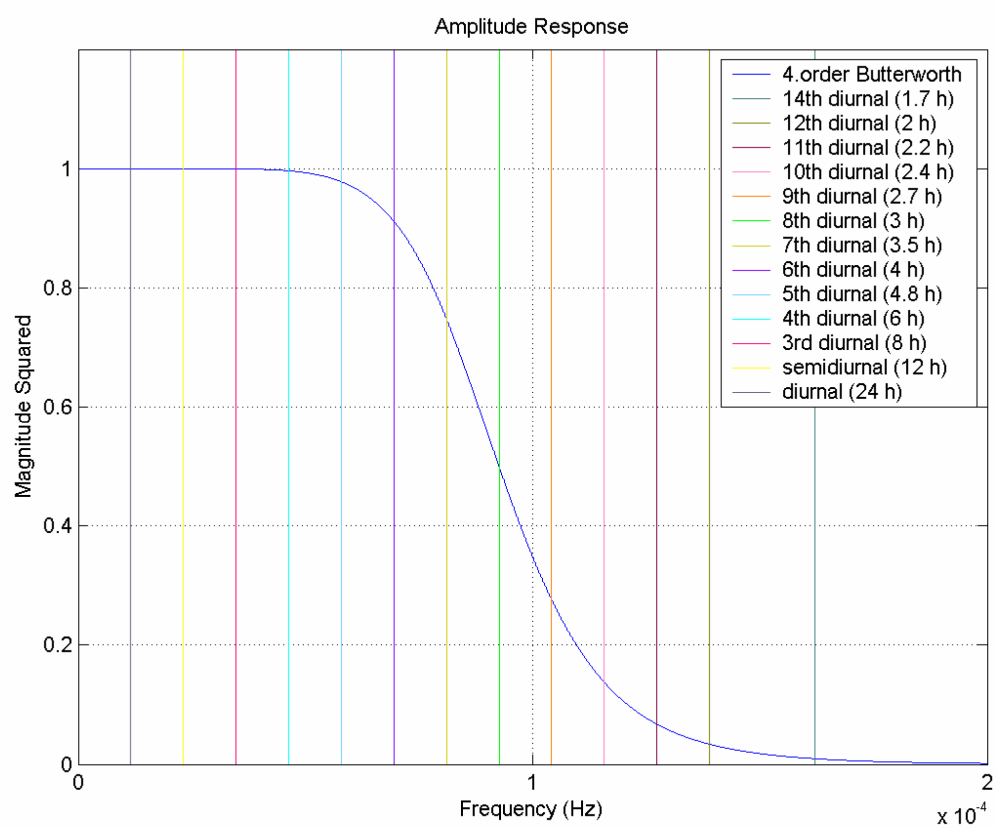

Figure 2-3 Squared amplitude response for a 4th order Butterworth LP filter with cut off frequency 1/(3 hours). Tide frequencies are shown. The Nyquist frequency for hourly sampling interval is 1/(2 hours), the same as the 12th diurnal frequency.

It is not meaningful to first attenuate these higher tidal frequencies, and then afterwards seeking the constant set for the same frequencies during the harmonic analysis. How much this will affect the resultant tidal model is beyond the scope of this note. Here we will limit ourselves to show a plot of the estimated power spectral density (PSD) of 1-minute water level data from Stavanger measured 05.09.03-23.10.03 as an example. The plot is shown in Figure 2-4, and we can clearly see peaks close to the 6th diurnal and 8th diurnal constituents. The Butterworth filter will attenuate these peaks, and the calculated amplitudes will be lower than they should have been for these constituents. Sites with shallow water effects (and then more over harmonic tidal frequencies,) will be most affected. This effect will be inspected more closely and documented in a new note.

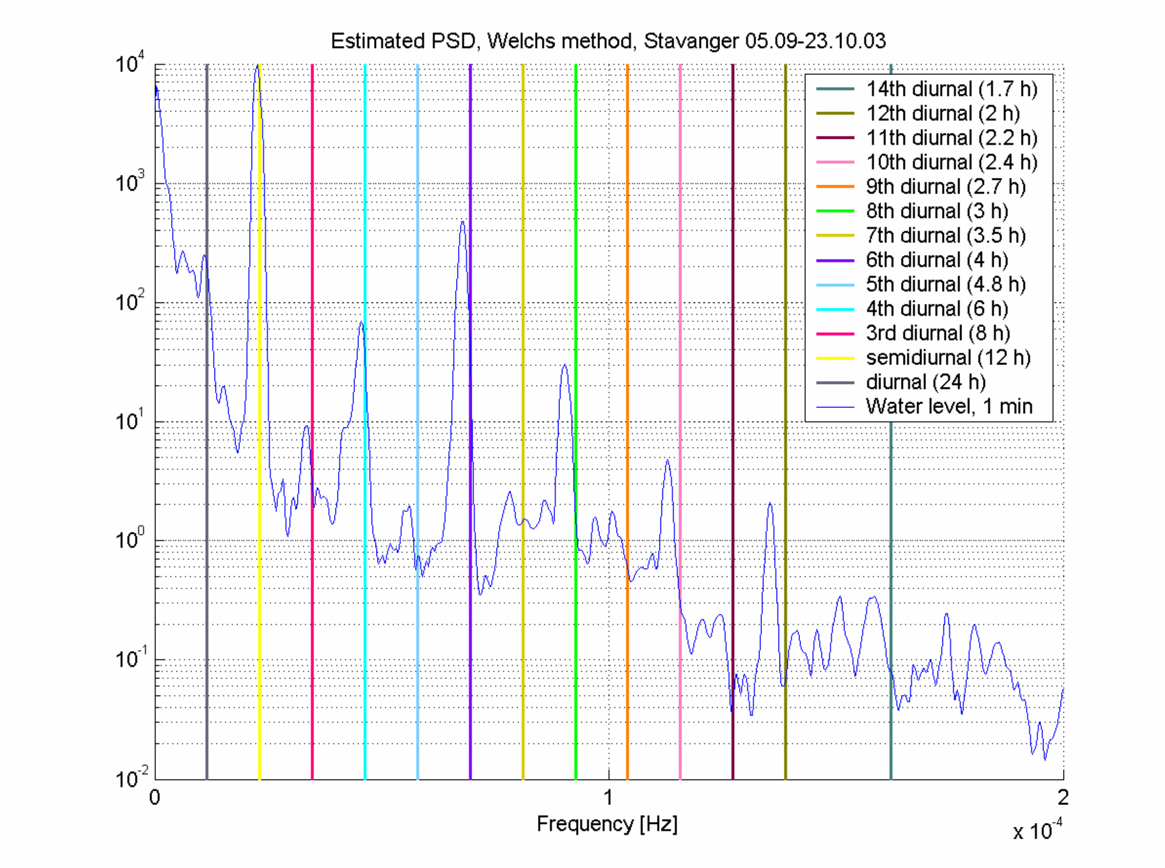

Figure 2-4 Estimated power spectral density of 1-minute water level data (pressure gauge inside a stilling well) and tidal frequencies.

An example of use of the filter is shown in Figure 2-5. 10-minute values from the water level gauge in Stavanger are shown together with the filtered version of the same time series. We can see that the filtered series is smoother.

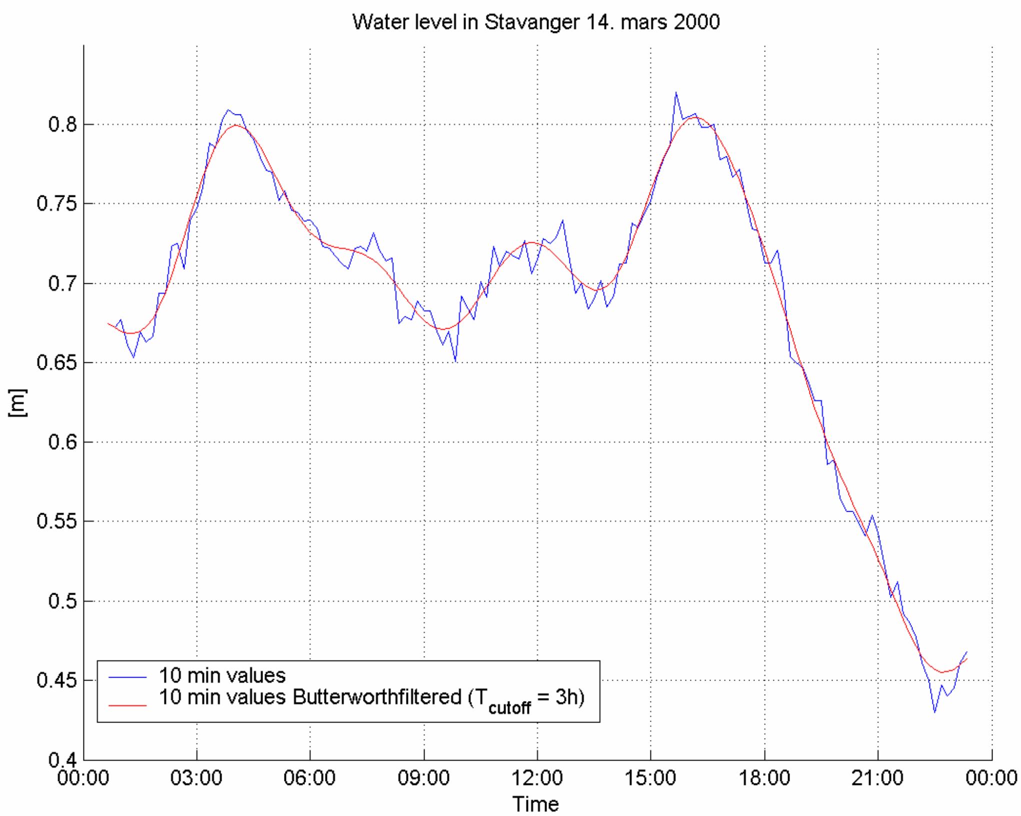

Figure 2-5 Measured water level in Stavanger: 10-minute values and Butterworth-filtered 10-minute values.

3 Filter 2: Smoothing filter

The purpose of this filter is to smooth the 10-minute values with an LP filter. The amplitude response is shown in Figure 3-1. The cut off frequency is 1/(1 hour).

The question is why is it necessary to smooth the 10-minute values? The reason is that the collected 10-minute values contain aliasing noise. The way water level data are collected and filtered before decimating to 10-minute values is described in detail in [8]. The conclusion from this report is that the filtering of the collected 1-second values is too weak. The filter used is 3 minute averaging, which has an amplitude response of –0.33 dB (= 20 log 0.963) at the Nyquist frequency for 10 minute sampling period.

$$f_{Nyq} = \frac{1}{2 \cdot 10 \cdot 60s} = 0{,}0008333\ldots \text{Hz} \qquad \text{(period 20 minutes)}$$

Waves with frequencies between the Nyquist frequency (0.000833…Hz) and 0.004 Hz is not dampen enough (period between 20 minutes and 4 minutes). The amount of aliasing noise will vary from site to site and with time (depending on the waves present at the site), and is not well known for our water level gauges. The highest frequencies in the 10-minute data series will be most exposed for aliasing noise. The purpose of the Butterworth filter is to dampen these high frequencies.

For most of our purposes these waves with high frequencies (period between circa 1 hour and 20 minutes) is not important. Actually for many purposes it can be important to remove these frequencies (even though they were not noisy). This is the case when the water level data is used on a site a little bit away from the water level gauges. For example by using the water level data for depth reduction in connection with measuring the seabed. The shorter period waves will not be equal at the water level gauge and at a site a bit away from the gauge.

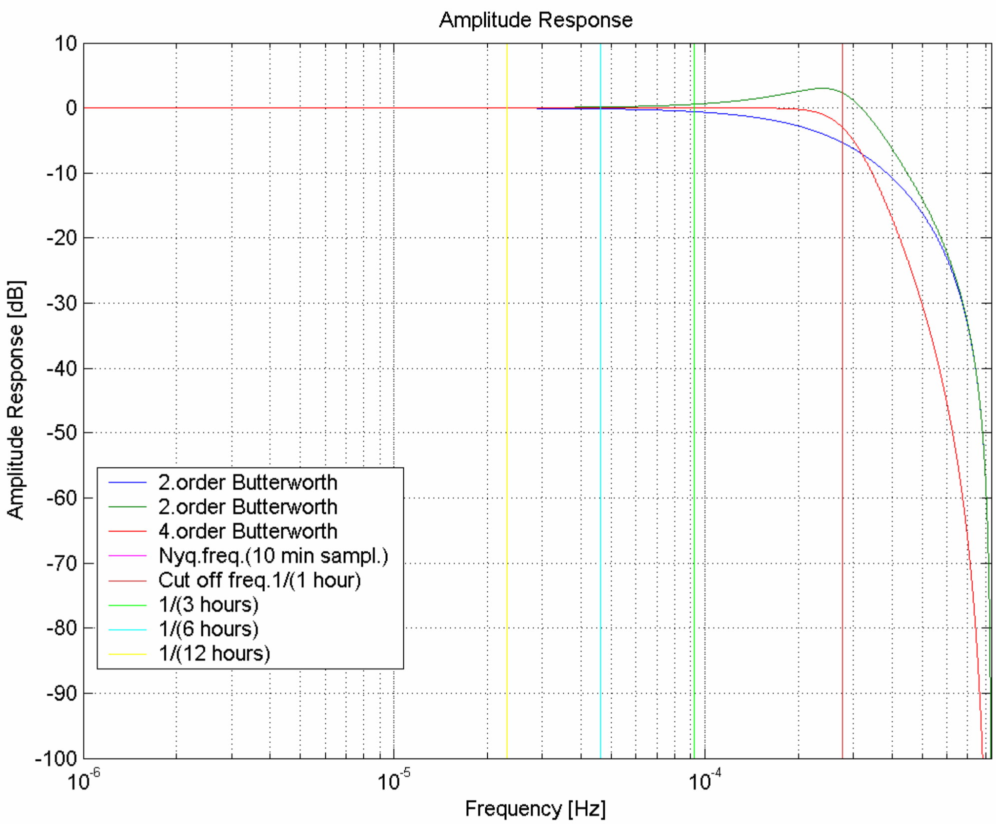

Figure 3-1 Amplitude response for a 4th order Butterworth LP filter consisting of 2 cascaded 2nd order filters. The Nyquist frequency for 10 minute sampling interval 1/(20 min) (almost invisible, since it coincide with the vertical right hand axis). The (-3 dB) cut off frequency 1/(1 hour) for the 4th order filter. Three of the tide frequencies 1/(3 hours) (8th diurnal constituents), 1/(6 hours) (4th diurnal constituents) and 1/(12 hours) (semidiurnal constituents).

Another plot of the amplitude response together with different tidal frequencies is shown in Figure 3-2. Here we see that the tidal frequencies is minimal affected by the filter.

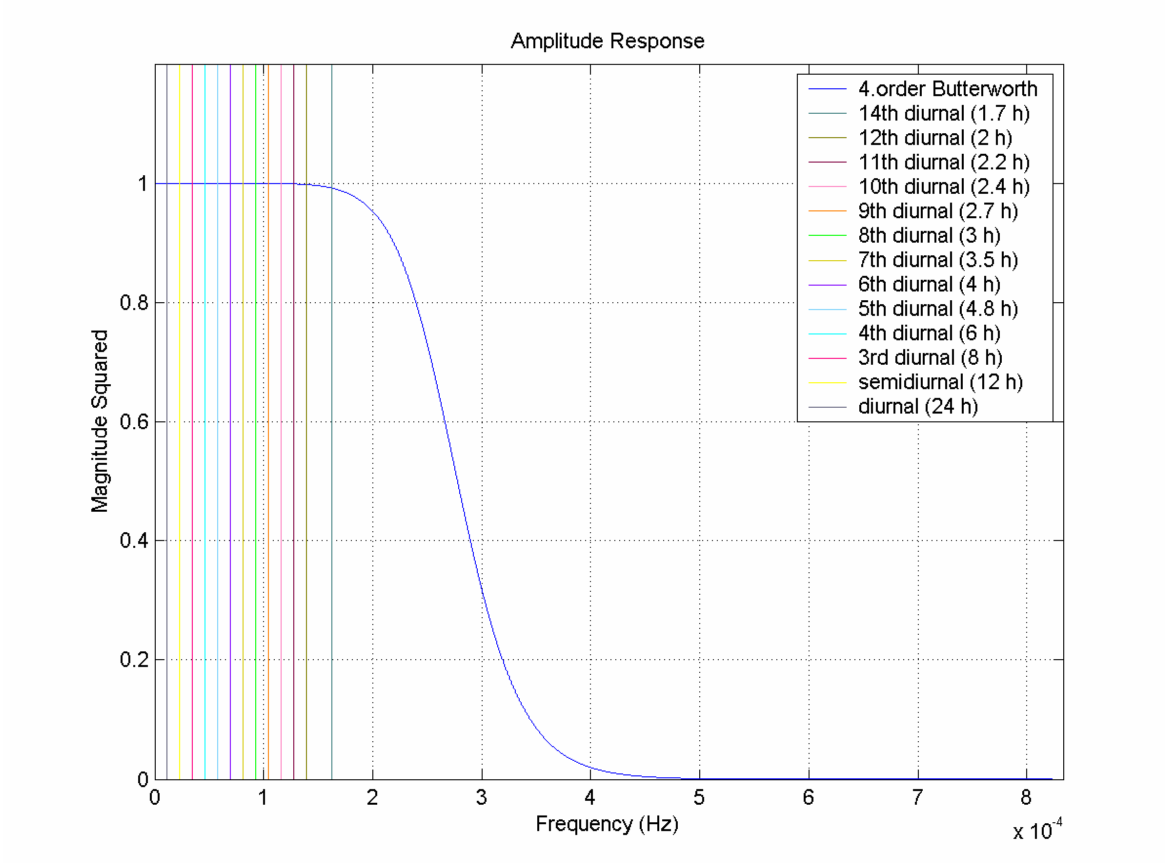

Figure 3-2 Squared amplitude response for a 4th order Butterworth LP filter with cut off frequency 1/(1 hours). Different tide frequencies are shown.

An example of use of the filter is shown in Figure 3-3. 10-minute values from the water level gauge in Stavanger are shown together with the filtered version of the same time series. We can see that the filtered series is smoother.

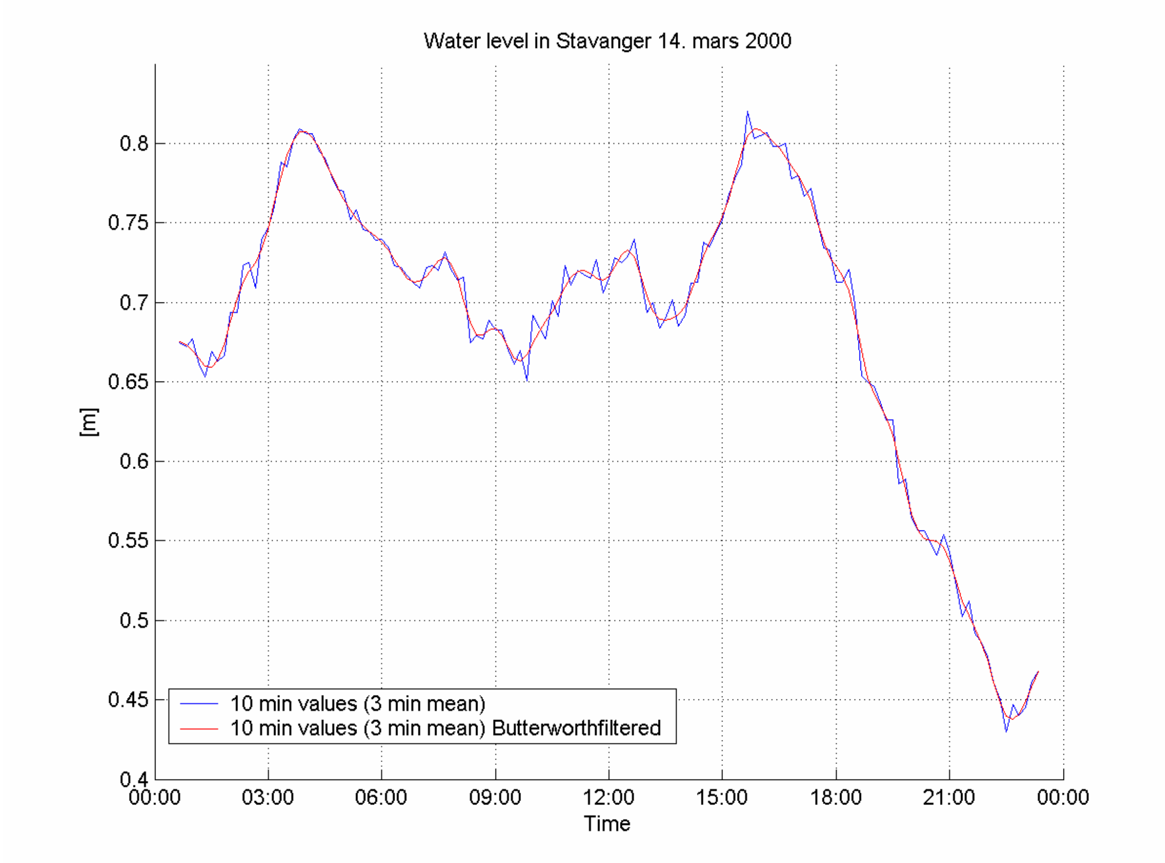

Figure 3-3. Measured water level in Stavanger: 10-minute values and Butterworth-filtered 10-minute values, cut off frequency 1/(1 hours).

In Figure 3-4 the Butterworth filtered 10-minute values are compared with 10-minute values collected with a more suitable filter than the 3-minute mean currently used. With this filter (FIR filter with order 3000) the 1-second time series is sufficient filtered before decimating to 10-minute values. And it is not necessary to filter the 10-minute values. See [8] for details.

We have an ongoing work looking at the possibility of changing the data-collecting scheme. This may remove the necessity of this smoothing Butterworth filter. This work will be documented in another note.

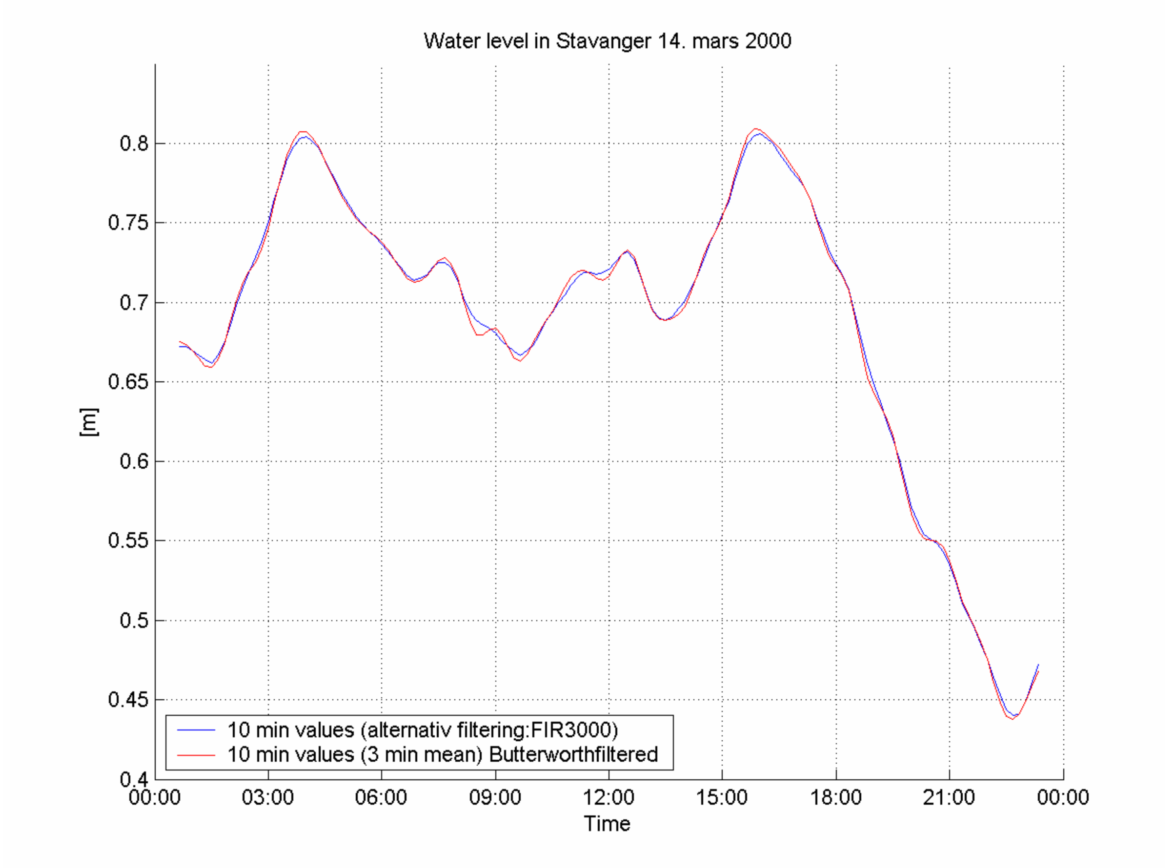

Figure 3-4 Butterworth-filtered 10-minute values compared with 10-minute values that are more correct collected (filtered with a FIR filter with order 3000 before decimating from 1-second values to 10-minute values see [8]).

References

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