

Sjøkartverket/Norwegian Hydrographic Service				Dok.id.: IV.2.7.2-1
Test of 30-sec sampling of water level and decimating to 10-min data.				Report
Versjon: 2.00	Gjelder fra: 19.04.2006	Utarbeidet av: Hanne Hodnesdal	Godkjent av: Hodnesdal, Hanne	Siden: 1 av 34

Test of 30-seconds sampling of water level and decimating to 10-minutes data.

Changed: 19.04.2006

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1 Abstract

A test series of water level data has been collected in Stavanger, Norway with a sampling period of 30 seconds. These data has been analysed both in time and frequency domain.

Low pass (LP) filters have been designed. These filters are anti-aliasing filters, which should be used before decimating from 30-seconds to 10-minutes values.

A moving average (MA) filter with order 20 (average over 10 minutes) is proposed for near real time (NRT) filtering. This filter will delay the data with 5 minutes. The delay requirements of NRT data are not yet decided. If the filter delay should be less than 5 minutes a lower order MA filter should be used, and if the filter delay can be longer than 5 minutes a longer FIR (finite impulse response) filter can be used.

The MA filter with order 20 could also be used for delayed mode (DM). However, a Butterworth filter run forward and backward has a much better amplitude response. Therefore a Butterworth filter with order 4 is proposed for delayed mode. The cut-off period must be between 50 minutes and 20 minutes, and in this report 50 minutes cut-off period is proposed.

In the case of the data set from Stavanger there is observed only a small difference between the decimated 10-minutes data that have been filtered by the two different low pass filters (MA order 20 and Butterworth order 4 with 50 minutes cut-off period).

2 Introduction

Norwegian Hydrographic Service (NHS) has permanent stations for sea level measurements called MDS. A MDS tide gauge is based on a float in a stilling well. The Norwegian Tide Gauge Network consists of 23 stations at different locations. This network was modernised in the period 2000-2002.

In [4] it is shown that the filtering and sampling scheme in the stations after the modernisation is not ideal. Specifically, the arithmetic mean over 3 minutes (which means mean of 181 data measurements, one measurement every second) every 10-minutes does not remove all energy above the Nyquist frequency before decimating to 10-minutes values. This means that the collected 10-minutes values contain aliasing noise. The effect on the data will vary with time and site, and is known best for the gauge in Stavanger where we have some test data.

It is waves with frequencies from 0.000833... Hz (period 20 minutes) and up to ca. 0.004 Hz (period ca. 4 minutes), which are not attenuated enough (by the 3-minutes mean filter), and consequently will appear as noise in lower frequency bands after decimating to 10-minutes values.

Since the datalogger used in the MDS only supports ordinary arithmetic mean, we can only choose the average period (length of mean interval, which is 3 minutes today) and sampling period (which is 10 minutes today).

In [4] the following sampling and filtering scheme was purposed as future sampling scheme:

Sample interval in encoder	1 second
Filtering in data logger	1 minute mean
Sample interval in data logger	1 minute
Filtering at office	LP-filter
Decimating at office	10 minute interval

Since then we have collected test series with 1-minute data. The analysis of these test series is documented in [6] and [7].

In the draft of the new [IOC Manual on Sea Level Measurement and Interpretation](#) [8] (page 4) it is recommended to sample at least once per minute or ideally as fast as once every 15 seconds if the water level gauge is supposed to be able to measure tsunamis. As a result of this recommendation we carried out a test with sampling every 30 seconds. The analysis of this test is documented in this report.

Sample interval in encoder	1 second
Filtering in data logger	29 seconds mean
Sample interval in data logger	30 seconds
Filtering at office	LP-filter (for near real time and delayed mode)
Decimating at office	10 minute interval

Note that [4] also concludes that the 1 Hz data measured in the stilling well in Stavanger which miss the narrowing at the orifice (stilling well and orifice has same dimensions), may contain aliasing noise (if there are waves with shorter period than 2 seconds). This is because this “stilling well” is not a low-pass filter in this case. This problem will not be addressed here.

3 Collecting test data

Test data were collected in the period 28.10-07.12.05 in Stavanger. In this period the float was measuring the water level with a sampling frequency of 1 Hz. The datalogger made an arithmetic mean every 30 seconds (an arithmetic mean of 29 1-second values), and stored the 30-seconds values. The test data series are shown in Figure 3-1.

It is a drawback that the test data were collected at the site in Stavanger, since it is a site that is missing the narrowing at the orifice. The characteristic of the data may not be representative for the data collected at the other sites. But it was the most practical site to use, since it is close to the office of Norwegian Hydrographic Service (NHS).

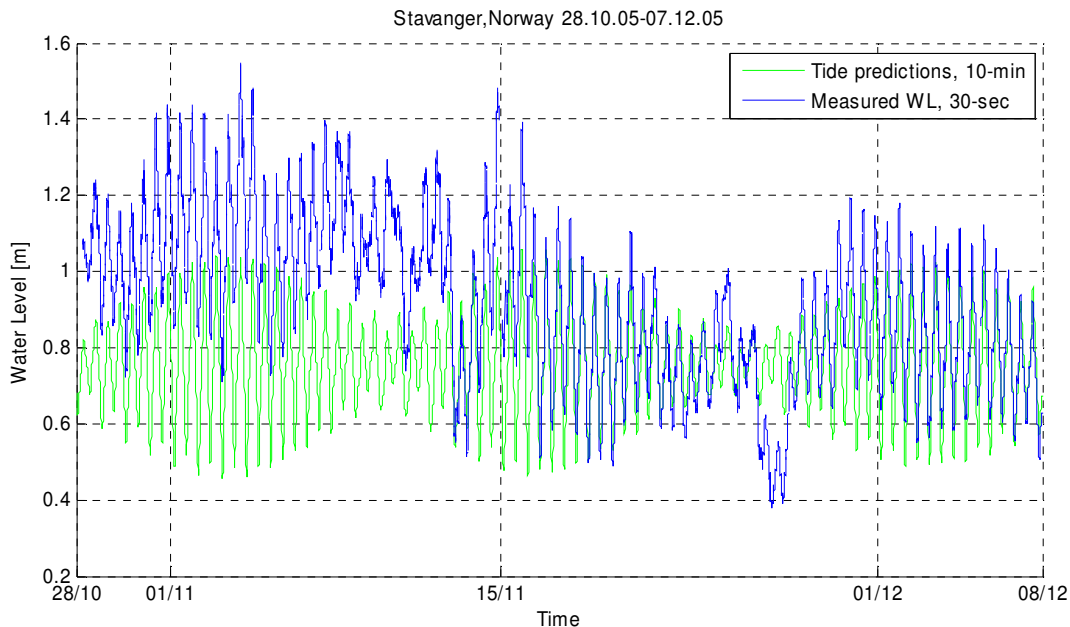


Figure 3-1 Measured water level (spikes have been removed) and predicted tide.

4 Analysis of test data

4.1 Spikes and gaps in the data series

There has been observed both some spikes and some gaps in the data set. The spikes are normally just before or after a gap (see Figure 4-1 - Figure 4-5). On 14 and 15 November there have been observed regular spikes/gaps once an hour (for example at 10:25:30 and 11:25:30). The spikes have been removed manually from the data set.

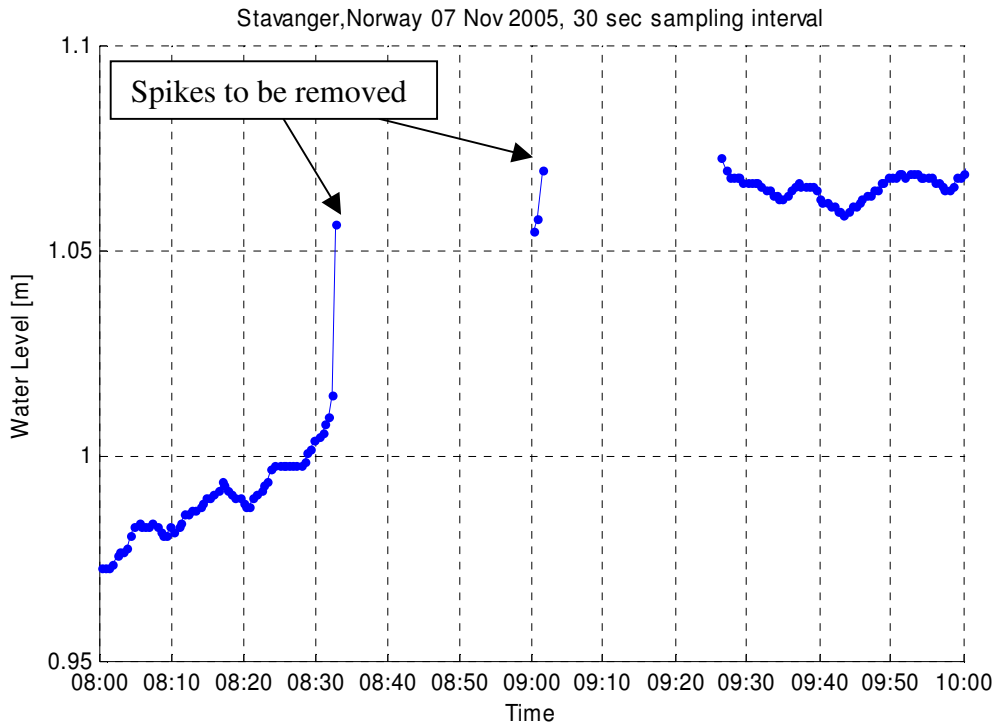


Figure 4-1 Two spikes on 7 November 2005.

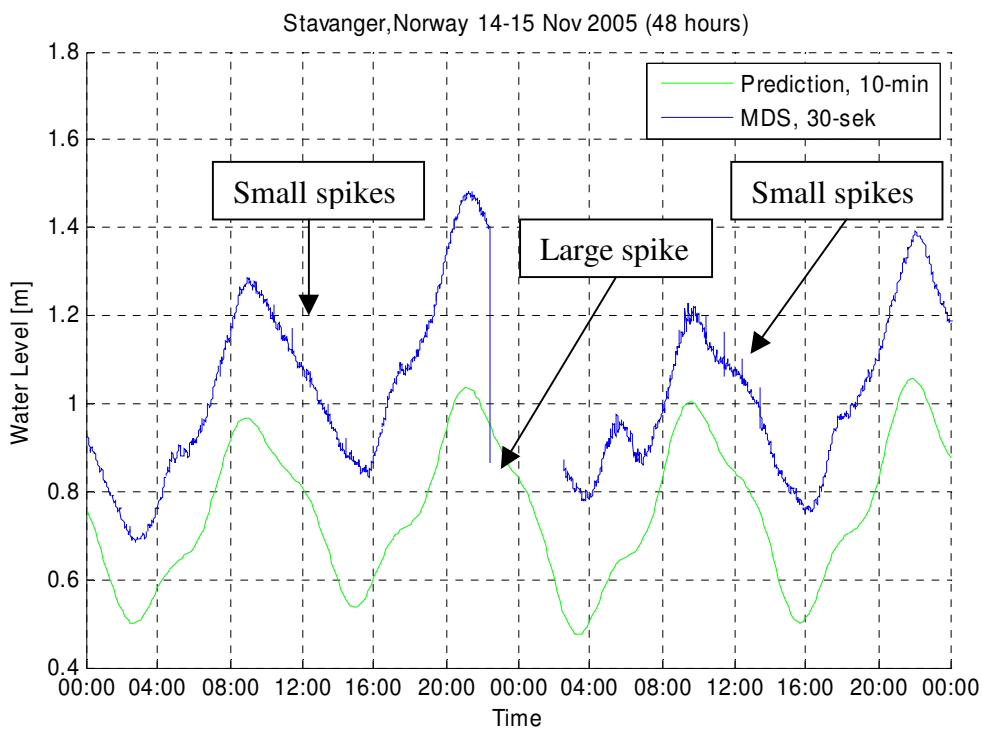


Figure 4-2 Spikes observed at 14 and 15 November 2005. The small spikes appear regular (about 25 minutes past a whole hour), see also Figure 4-3 and Figure 4-4

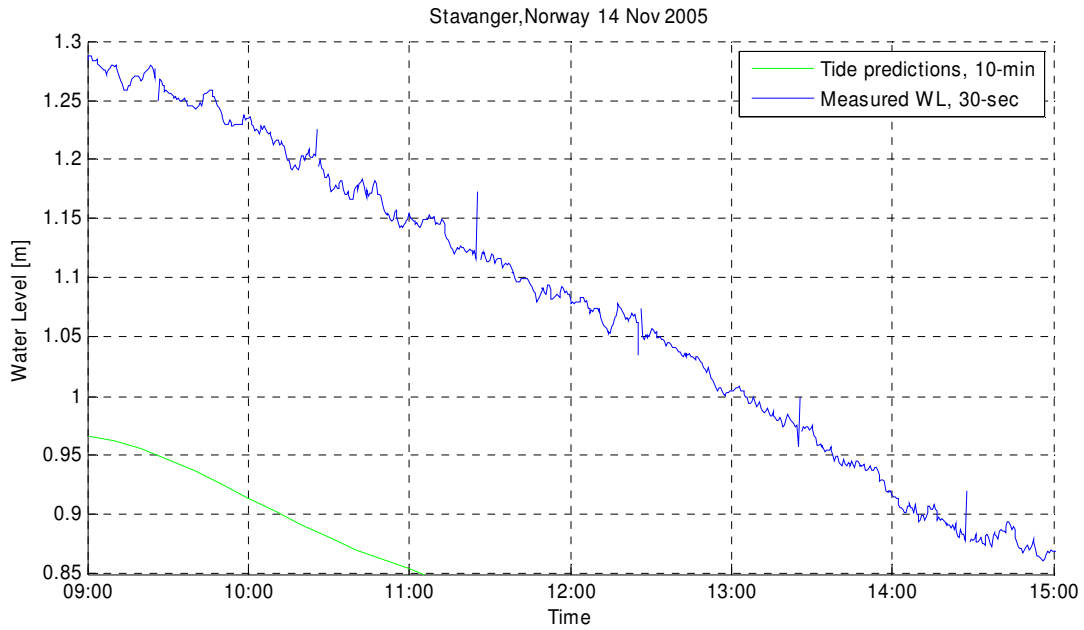


Figure 4-3 Spikes observed on 14 November 2005. The small spikes appear regular.

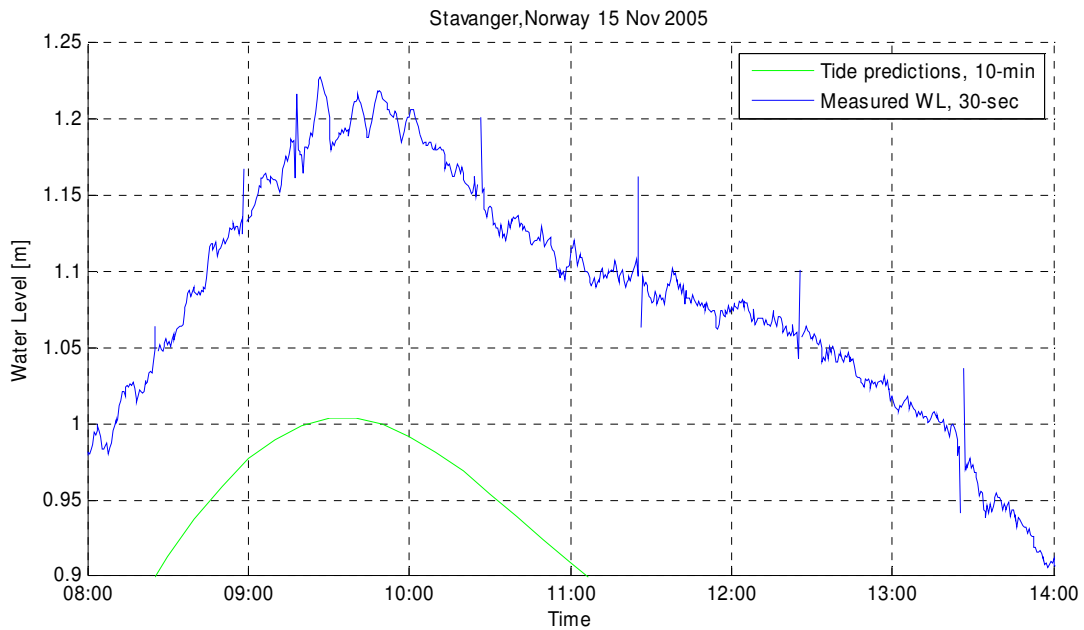


Figure 4-4 Spikes observed on 15 November 2005. The small spikes appear regular

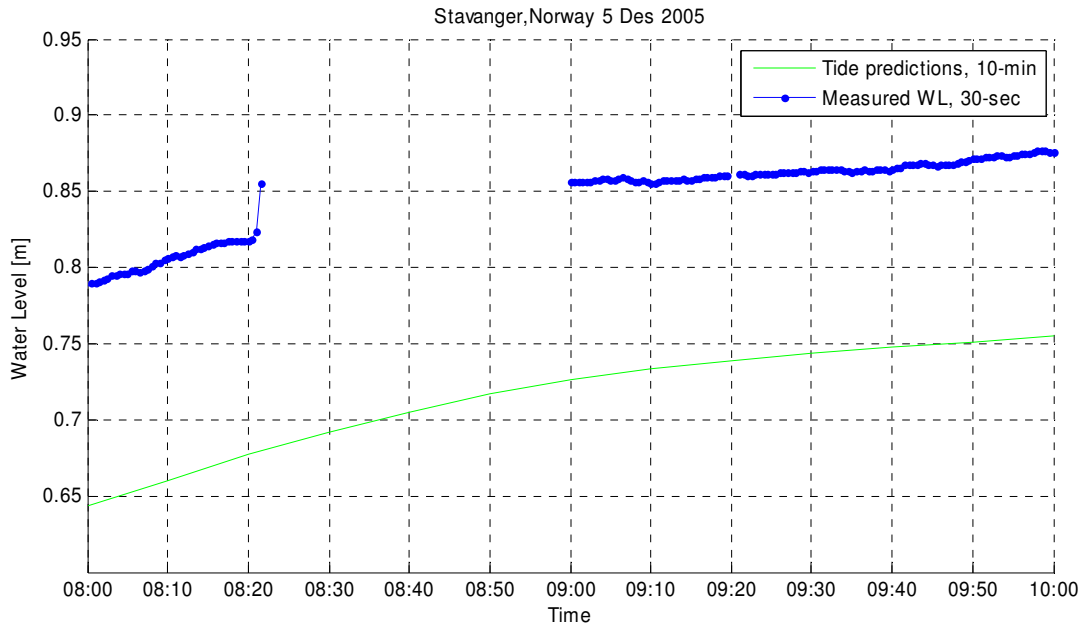


Figure 4-5 Spike observed on 5 December 2005.

4.2 Interpolation

It is observed 396 gaps in the data set in this test period. In 314 of these cases there is only missing one 30-second sample, which means the gaps are one minute long. In 51 of the cases it is missing two samples (the gaps are 1.5 minutes). In 14 of the gaps it is missing three samples (the gaps are two minutes) and in 5 of the gaps it is missing four samples (the gaps are 2.5 minutes). The rest of the gaps, totally 12, are longer gaps from 4 minutes to 246 minutes (4:06 hours).

Gaps longer than 10 minutes are filled with predictions adjusted to the water level data (6 gaps are longer than 10 minutes). Smaller gaps are filled with linear interpolation. The resulting data set is shown in Figure 4-6 (whole data series) and Figure 4-7 (48 hours).

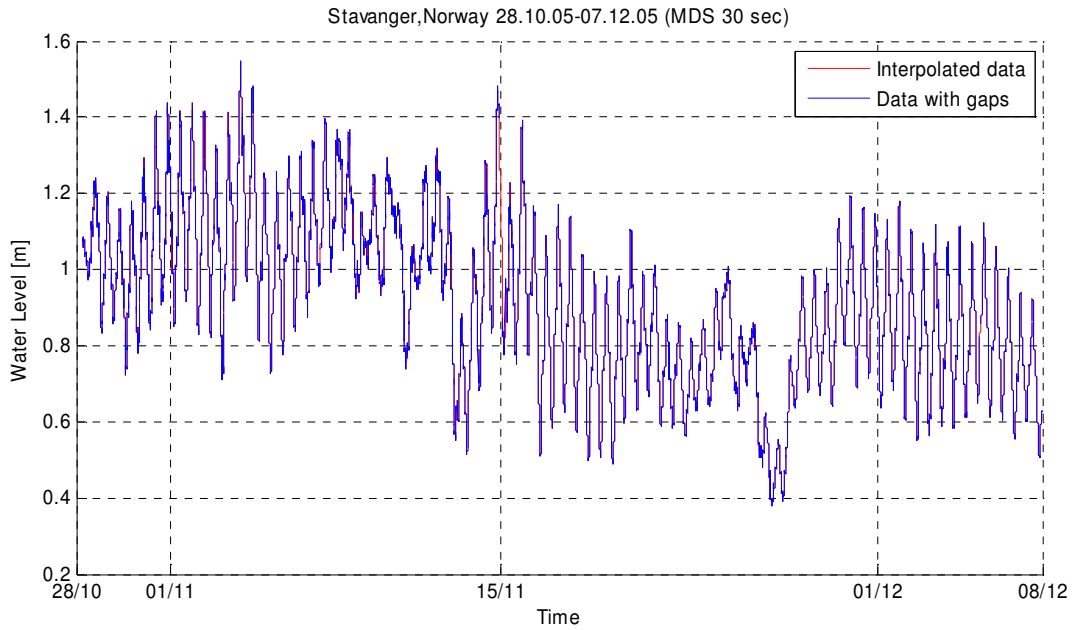


Figure 4-6 Data set with gaps and data set where the gaps are filled with predictions (gaps > 10 minutes) and interpolated data (gaps < 10 minutes).

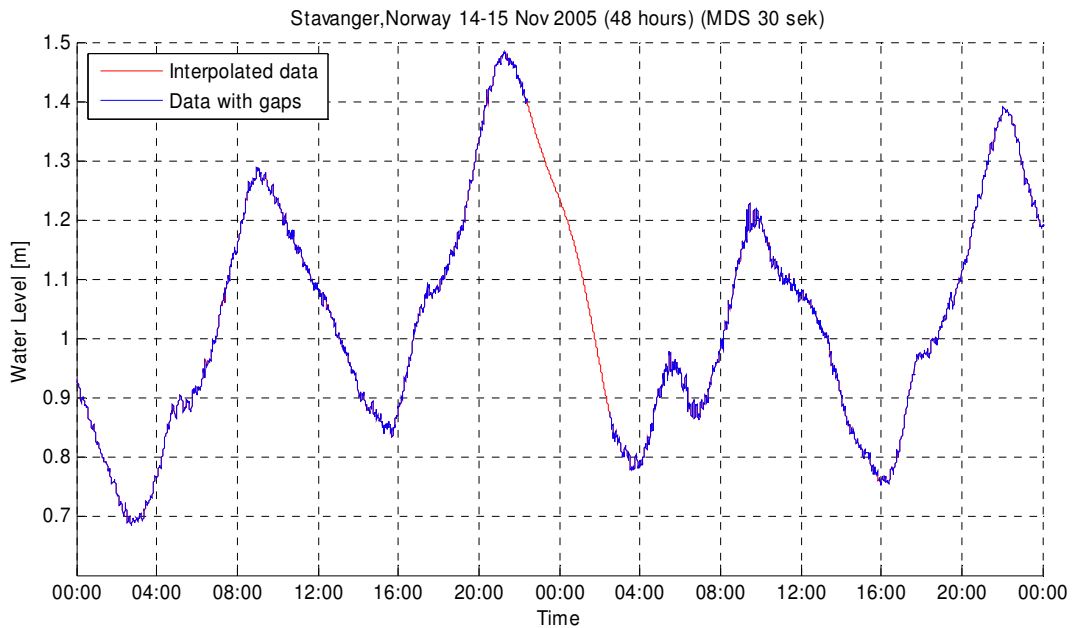


Figure 4-7 Data set with gaps and data set where the gaps are filled with predictions (gaps > 10 minutes) and interpolated data (gaps < 10 minutes).

4.3 Frequency content

The power spectral density (PSD) of the test data series is estimated and shown in Figure 4-8 (whole PSD) and Figure 4-9 (segment of the PSD). Welch’s method is used in computing the PSD. This method makes a compromise between resolution and the variance of the estimate. With a longer time series we can get higher resolution and/or lower variance of the estimate.

We can see that the PSD has peaks at the tidal frequencies (which are marked with vertical coloured lines). The powers at the higher frequencies (above the tidal frequencies) will be mainly wind driven (see [1] for a discussion of wave forming sources).

Note that the measured water level is not a stationary time series. The frequency content is weather dependent, and therefore time dependent (since weather changes with time). The estimated PSD in Figure 4-8 and Figure 4-9 shows the overall PSD for the whole period of the data set. However in Figure 4-12 and Figure 4-13 the PSD for two different periods are compared. Both periods are 24 hours long. The first period is 8 November 2005 which is a day with quite turbulent water level (see plot of time series in Figure 4-10). The second period is 29 November 2005 where the sea level is calmer (see plot of time series in Figure 4-11).

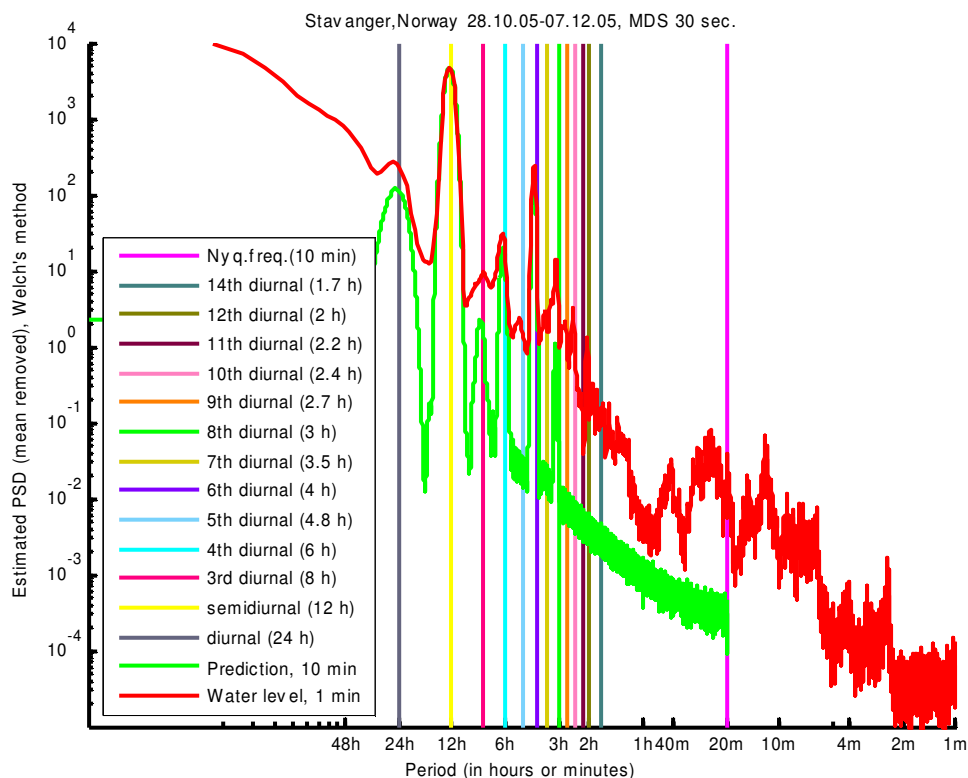


Figure 4-8 Estimated PSD of 30-second water level data and 10-minutestide predictions.

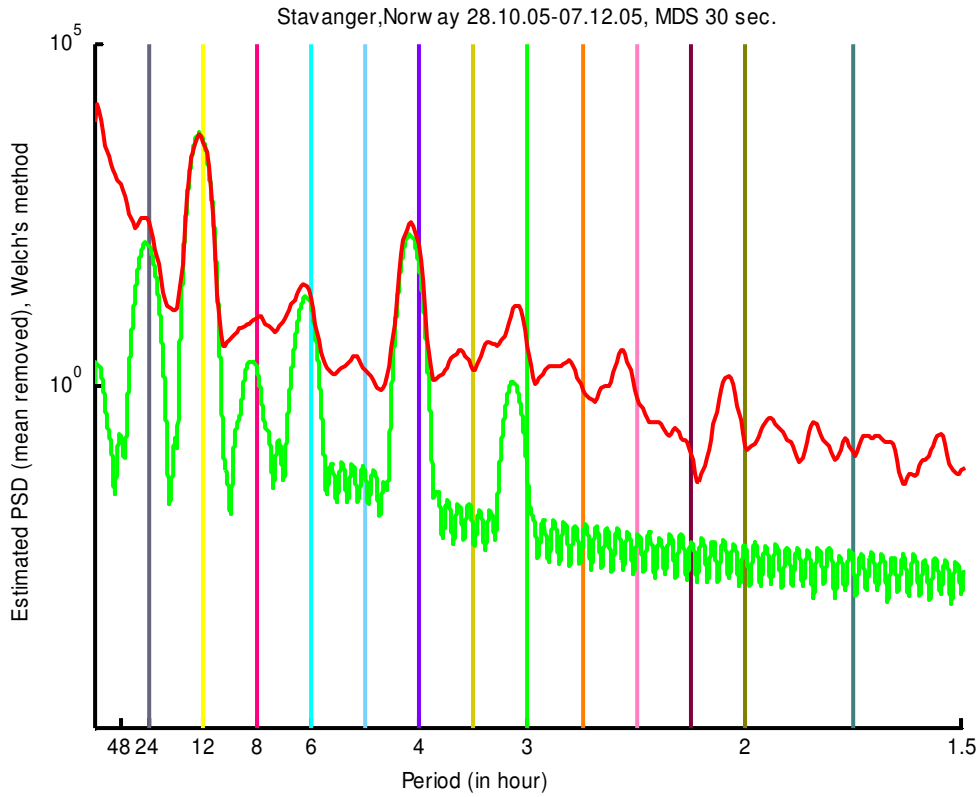


Figure 4-9. Same as Figure 4-8, but here it is focused on the tidal frequency band.

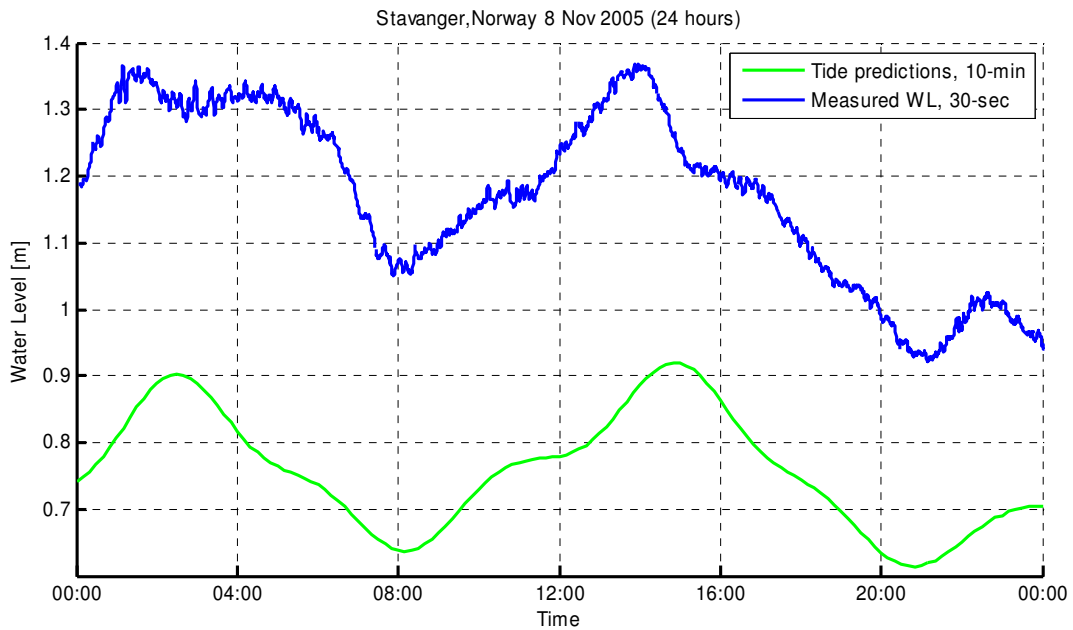


Figure 4-10 Measured water level and predicted tide.

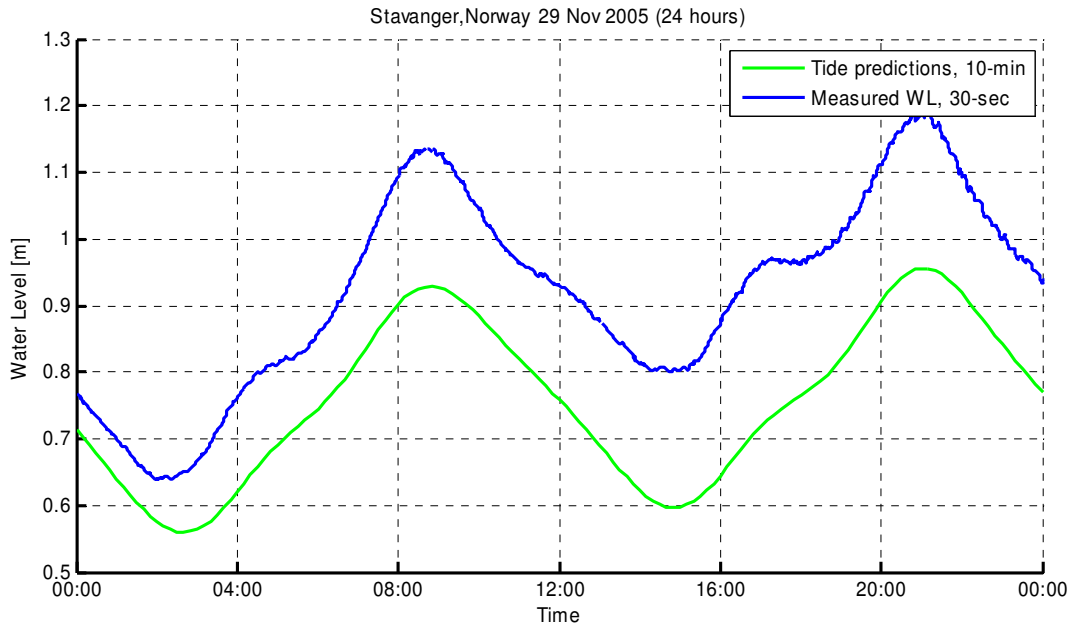


Figure 4-11 Measured water level and predicted tide.

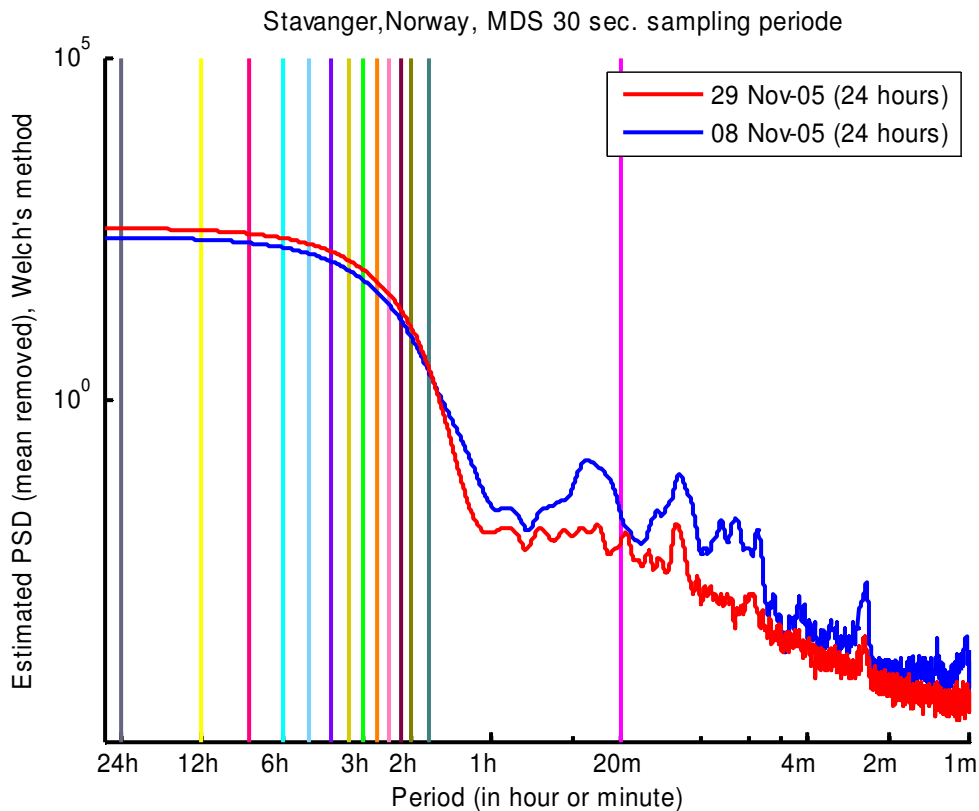


Figure 4-12 Estimated PSD of 30-second water level data from two different periods.

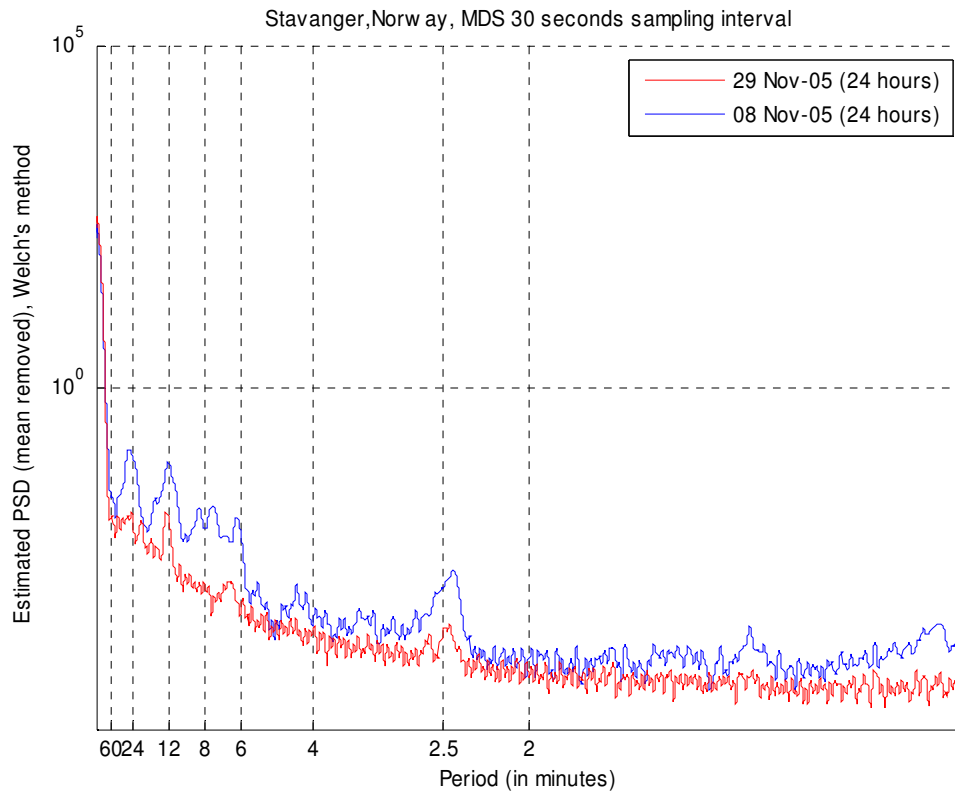


Figure 4-13 Same as Figure 4-12 but focusing on higher frequencies.

5 Filtering and decimating

5.1 Design of low pass filter

Normally we are not interested in the waves with the highest frequencies. For example when the data are supposed to be used for a whole area, and not just at the point where the gauge is placed. The high frequency components will normally be very local. Another example is when the data is used for harmonic analysis, then we are only interested in the tidal frequencies and their over-harmonics. For these reasons the water level database are based on 10-minutes values (harmonic analysis is based on 1-hour values). Before decimating to 10-minutes values the 30-seconds series has to be filtered with an anti-aliasing filter (low pass filter) to remove frequencies above the Nyquist frequency for 10-minutes values. (See more about anti-aliasing filter and Nyquist frequency in [4].)

When we decimate from 30-seconds values to 10-minutes values (decimate from 0.0333... Hz to $f_{s_dec} = 0.00166...$ Hz), all frequencies above f_{max} should be removed.

$$f_{max} \leq \frac{f_{s_dec}}{2} = \frac{1}{2 \cdot 10 \cdot 60s} = 0,0008333... \text{ Hz}$$

At the same time the tidal frequencies and their over-harmonics should be as undisturbed as possible. This sets the requirements for the filter:

- The pass-band start at 0 Hz and should end at about $1/(1 \text{ hour}) = 0.0002777... \text{ Hz}$
- The stop-band should start at $1/(20 \text{ minutes}) = 0.0008333... \text{ Hz}$ and end at $f_s/2 = 1/(30 \text{ sec} \cdot 2) = 0.01666... \text{ Hz}$ (Nyquist frequency for 30 seconds sampling period)

The end of the pass-band is chosen in such a way that the tidal frequencies and their over-harmonics will not be disturbed (attenuated). The 14th diurnal component has a period of 1.7-hour (see Figure 4-9), which means that the frequency is 0.0001634 Hz and will be in the pass-band. Higher tidal over-harmonics frequencies are anticipated to be of little importance.

However, if we want to keep more of the higher frequency signals ($f > 1/(1 \text{ hour})$) in the 10-minutes values then the end of the passband should be set to a higher frequency. The theoretical limit for how high frequencies the 10-minutes data can contain is $1/(20 \text{ minutes})$. But since it is not possible to use an ideal filter the end of the passband should be chosen lower than $1/(20 \text{ minutes})$. Consequently, it is *possible to choose the passband to end somewhere between 1(1 hour) and 1(20 minutes)*. The closer to $1/(20 \text{ minutes})$ the passband ends the steeper the slope of the filter has to be. For our applications the 10-minutes values are supposed to be used for low frequency phenomena (tidal frequency over-harmonics and lower frequencies) and therefore *we choose the end of the pass-band at 1/(1 hour)*.

In any case (if the end of the passband is set to $1/(1 \text{ hour})$ or somewhere closer to $1/(20 \text{ minutes})$) the frequencies in question, with period between 1 hour and 20 minutes, can always be available from the original 30-seconds data which will be preserved.

5.1.1 Near real time filter

If the 10-minutes water level data are supposed to be published in near real time (NRT), the filter should not cause too much delay. In addition the filter can not be run forward and backward (which is only possible in a post-processing mode). As a consequence the NRT filter should be a relatively short (depending on the delay requirements of the NRT application) FIR filter.

The delay requirements of the NRT data are not decided yet. In this document we anticipate that the filter should not delay the data with more than 5 minutes. (There are other sources for delay as well: Filtering in the data logger, transferring data from the data logger to the database and automatic quality control of the data before filtering. These are not addressed here)

With 5 minutes delay, the filter can use 10 minutes of data when calculating the output (5 minutes before and 5 minutes after the time instant). For example a moving average (MA) filter can be used and the output at for example time 10:00:00 is calculated as a mean of the values at [09:55:00, ..., 09:59:30, 10:00:00, 10:00:30, ..., 10:05:00]. Total 21 input values are used to calculate 1 output value, which means that the filter has order 20.

We can also design a 20 order LP FIR-filter in Matlab as shown in Figure 5-1 where we have specified the cut-off frequency $F_{c_{Hz}}$ to be $1/(20 \times 60)$. However, the window method is unreliable in this case because the order is low (only 20) and the specified normalised cut-off frequency $F_{c_{Norm}}$ is close to the zero frequency. Specified normalised cut-off frequency¹ is in this case:

$$F_{c_{Norm}} = \frac{F_{c_{Hz}}}{F_s/2} = \frac{1/(20 \cdot 60)}{1/60} = \frac{1}{20} = 0.05$$

We can see in Figure 5-1 that although F_c (-6 dB) in this case is specified to be $1/(20 \times 60)$ Hz = 0.8333... mHz (period 20 min) the actual designed filter has an amplitude response with $F_c \approx 1.58$ mHz (period 10.5 min).

¹ Note that when using the window design method the cutoff frequency of the filter is the -6 dB frequency.

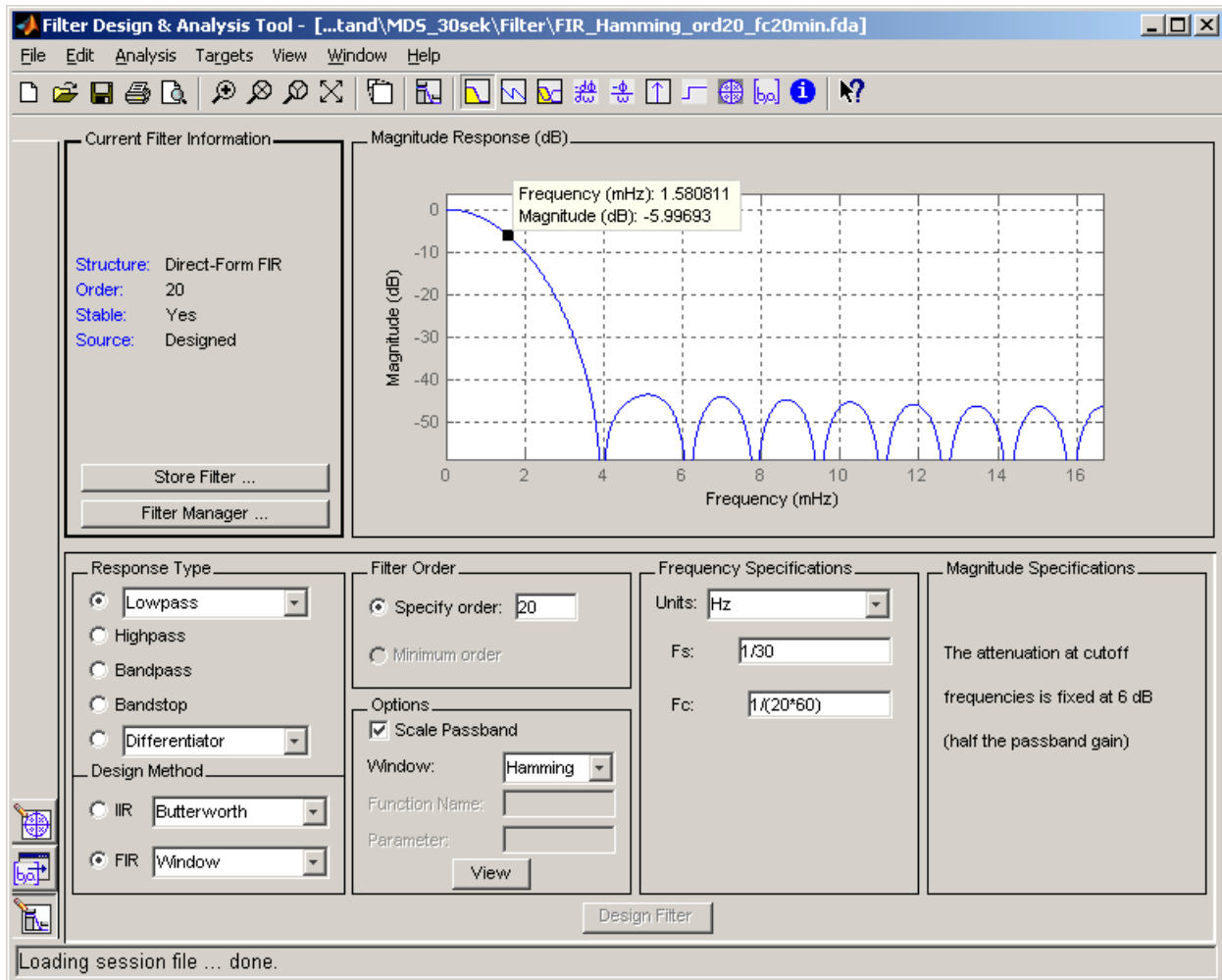


Figure 5-1 Design of LP FIR filter in Matlab using the window method. It is not possible to achieve the specified cutoff frequency with the chosen filter order.

The amplitude responses of 5 different low pass FIR filters are shown in Figure 5-2 - Figure 5-4. Three of the filters are of order 20, which means that the delay is 5 minutes. These three filters are one moving average and two window-designed filters (Hamming window) that are designed with different -6 dB cutoff period (namely 20 minutes and 50 minutes). However the specified cutoff is not achieved. The two other filters are shorter moving average filters of order 10 and 6 (which means delay of 2.5 and 1.5 minutes).

MA filters in general has a slow roll off and bad stopband attenuation [9]. This can be observed in Figure 5-2 and Figure 5-3. The stopband is actually only effective for discrete frequencies. These discrete frequencies are multiple of $1/(\text{filter length})$ [10] and in the case of the MA filter with order 20 they will be: $f_s/21$, $2x f_s/21$, $3x f_s/21$... which corresponds circa to the periods (in minutes): 10.5, 5.25, 3.5,...

The shorter MA filters attenuate less than the MA filter of order 20 and they do not attenuate enough the signals above the Nyquist frequency. These shorter filters are only recommended if the filter delay requirement is stricter than 5 minutes and the aliasing noise is not so important.

The window-designed filters (Hamming window) of order 20 have much better stopband attenuation than the MA filters (see Figure 5-2 and Figure 5-3). However, since the chosen order was too small to get the specified cutoff frequency (the cutoff frequencies of the filters

are higher than specified), these filters are not good either. However, if we tolerate a longer delay we can choose a higher order for the window designed filters. For example order 60 (15 minutes delay) will give the two window-designed (Hamming) filters in Figure 5-5 and Figure 5-6. Now we can see that the cutoff frequencies (-6 dB) are more as specified. Stopband attenuation is good, but roll off is still quite slow.

In Figure 5-4 and Figure 5-6 (which both focus on the passband) we can see that the MA filter of order 20 starts to attenuate the highest tide frequencies. However, the amount is small. The window-designed Hamming filter of order 60 with $t_c = 20$ min attenuates the tide frequencies about the same amount as the MA filter of order 20, while the window-designed Hamming filter of order 60 with $t_c = 50$ min attenuates more.

When we compare the filters that have been analysed in this section we can see that the window-designed Hamming filter of order 60 with $t_c = 20$ min has the best overall filter characteristics for an anti-aliasing filter. However the delay is 15 minutes. If delay requirement is set to 5 minutes the MA of order 20 is the best anti-aliasing filter.

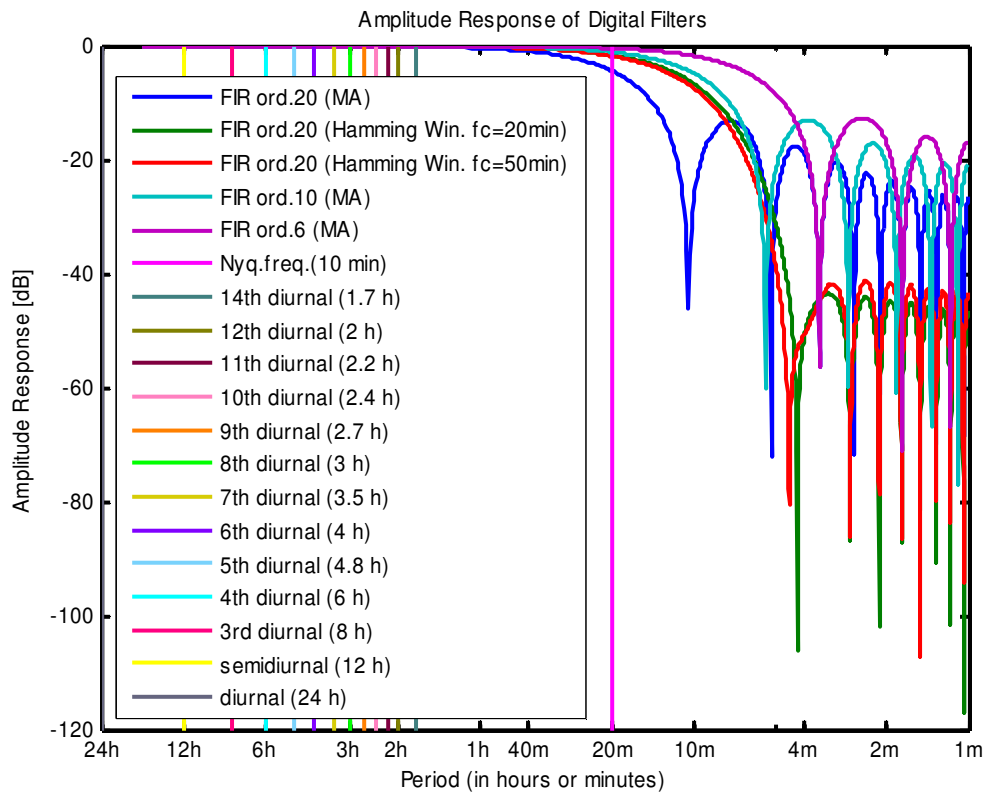


Figure 5-2 Amplitude responses of 5 different FIR filters (Hamming and MA)

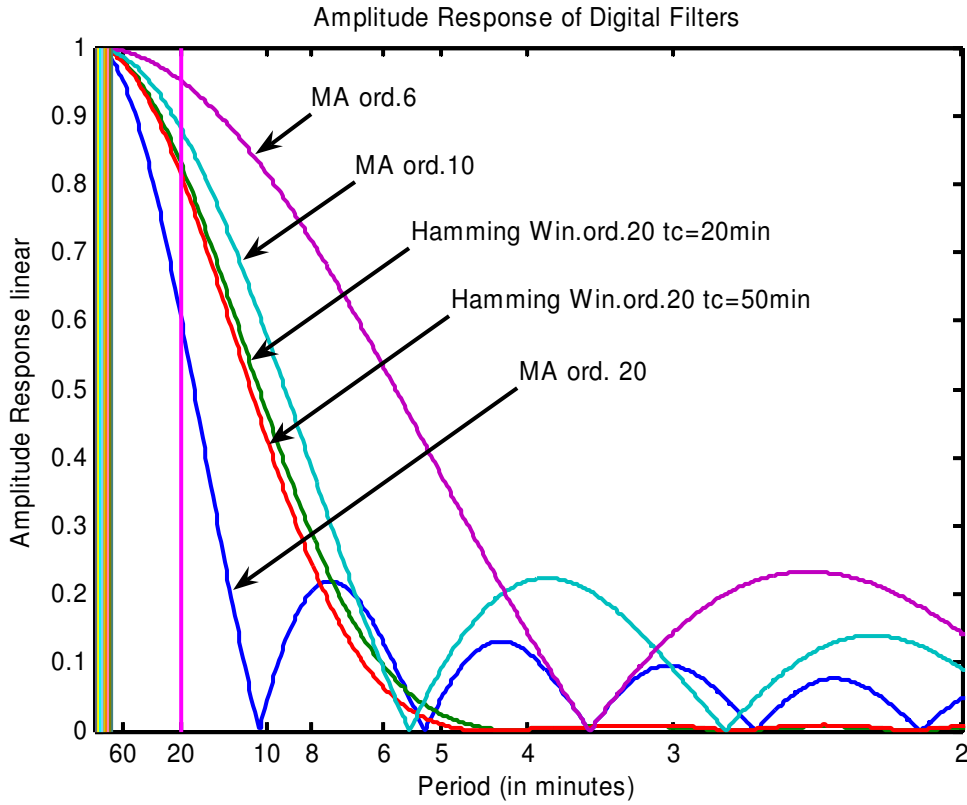


Figure 5-3 Amplitude responses of 5 different FIR filters (Hamming and MA)

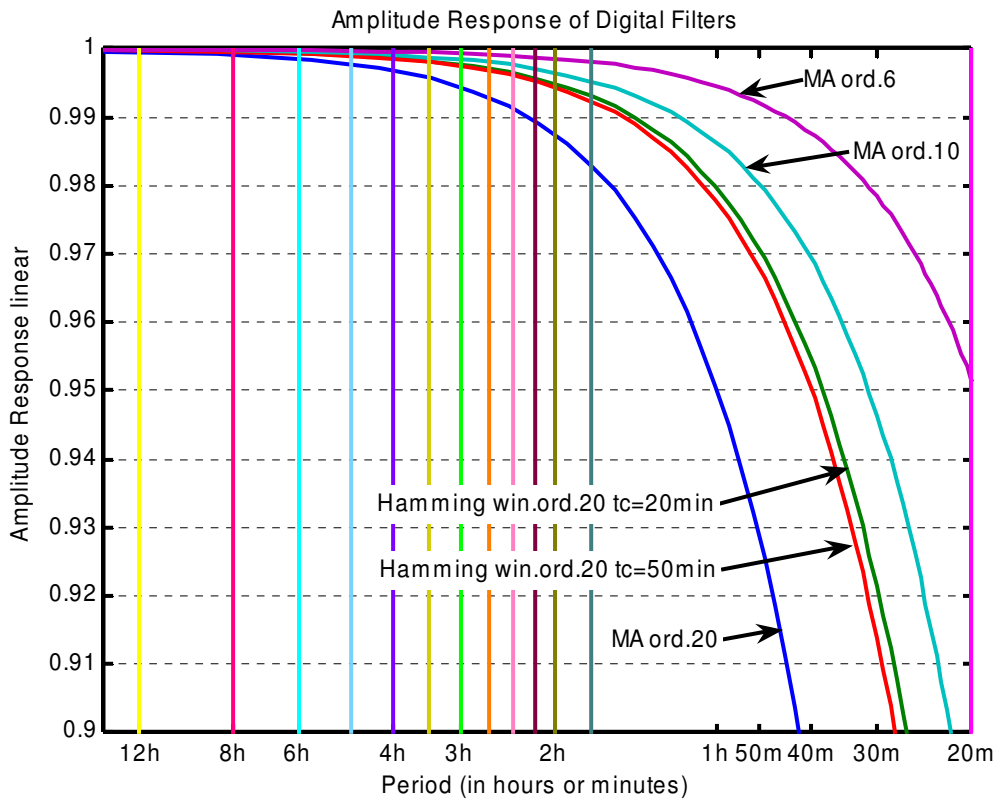


Figure 5-4 Amplitude responses of 5 different FIR filters (Hamming and MA)

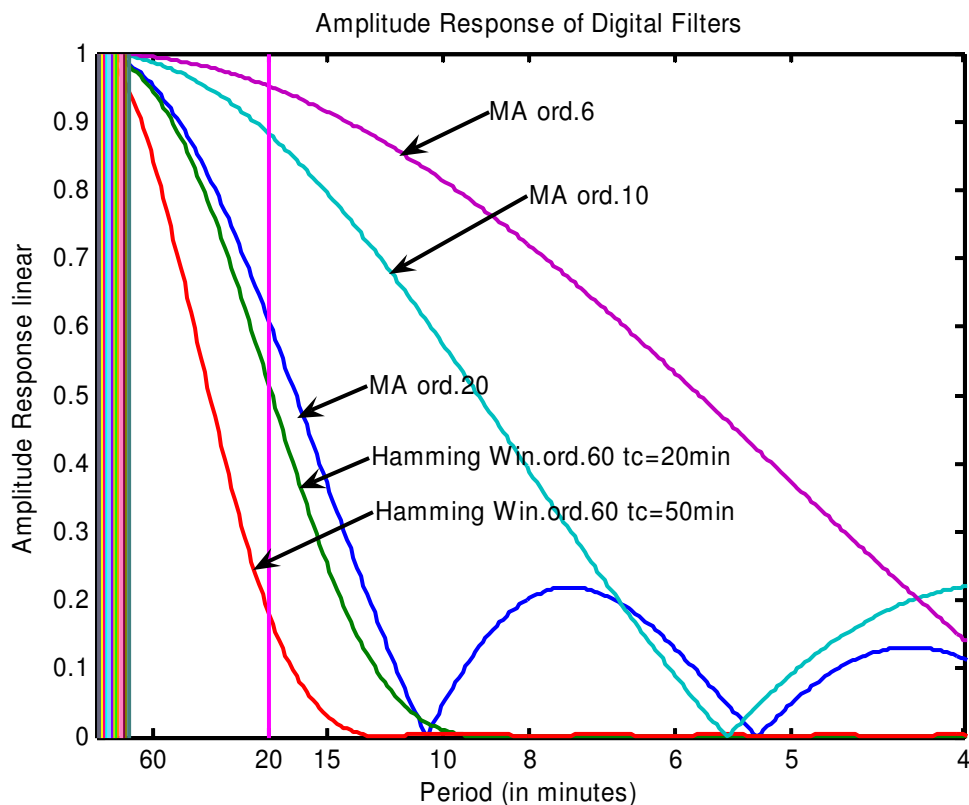


Figure 5-5 Amplitude responses of 5 different FIR filters (Hamming and MA)

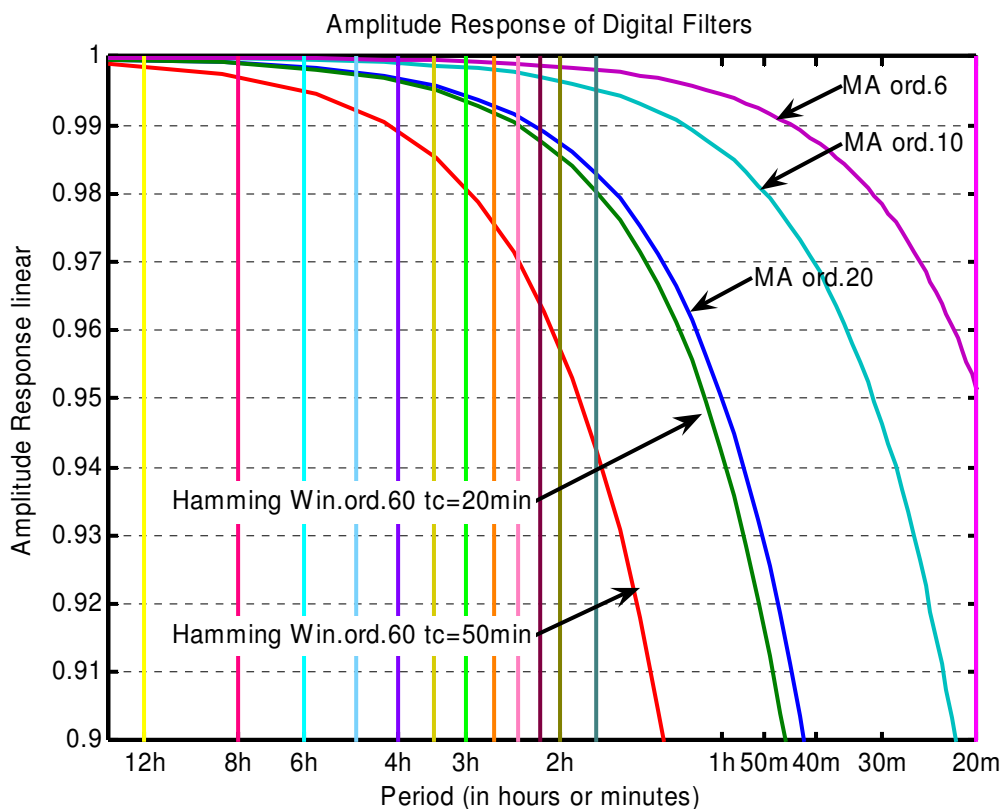


Figure 5-6 Amplitude responses of 5 different FIR filters (Hamming and MA)

5.1.2 Delayed mode filter

Since the DM filter may be used at the office in a post-processing mode, we can filter both forward and backward in time to get a zero phase filter response. This means that an IIR²-filter can be used. A Butterworth-filter will be a good choice since it has a magnitude response that is maximally flat in the pass-band and monotonic overall.

Figure 5-7 shows a design of a Butterworth-filter using Matlab.

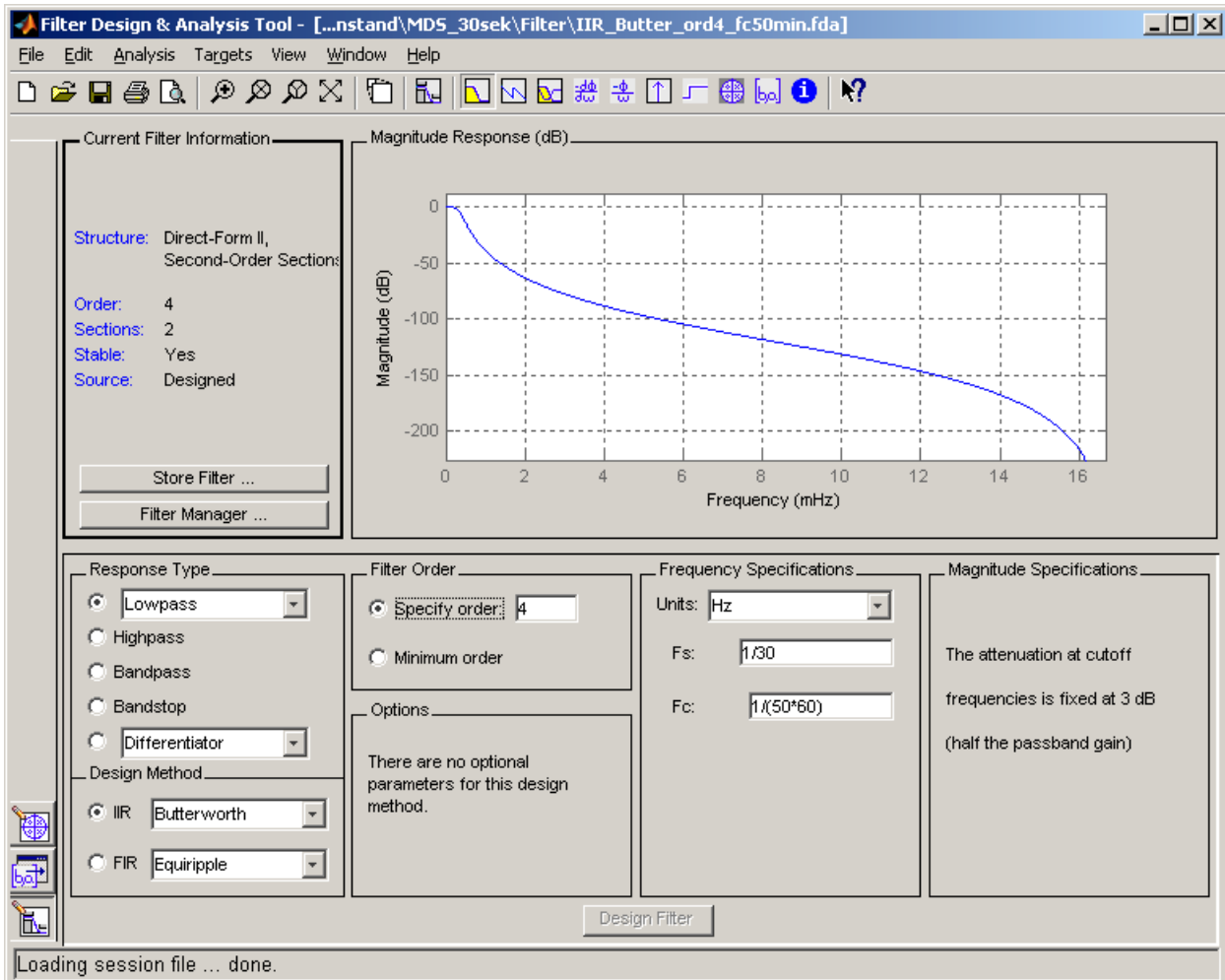


Figure 5-7 Design of low pass Butterworth filter with order 4 in Matlab

The amplitude responses of 5 filters are shown in Figure 5-8. Four different Butterworth filters and a FIR filter (MA order 6) are compared. In Figure 5-9 the same four Butterworth filters are compared with a longer FIR-filter (MA order 20). We can see that the Butterworth filters of order 6 are steeper than the Butterworth filters with order 4, which in turn is *much* steeper than the MA filters (even if the MA filter is longer).

The Butterworth filters attenuates the tide frequencies, which are in the passband, less than the MA filter of order 20 (visible in Figure 5-9). However, as mentioned in section 5.1.1, the attenuation is small even for the MA of order 20.

The Butterworth filters will be used both forward and backward. This means that the total amplitude response will be the squared amplitude response, which is even steeper. The squared amplitude responses of the four Butterworth filters are shown in Figure 5-10 and Figure 5-11.

² IIR = Infinite Impulse Response

Two different cut off frequencies (-3 dB) are used when designing the Butterworth filters, namely $f_c = 1/(50 \times 60)$ Hz and $f_c = 1/(20 \times 60)$ Hz, which means that the cut off period is $t_c = 50$ minutes and $t_c = 20$ minutes respectively. In the case of 10-minutes values we are interested in tide frequencies and other slowly varying phenomena. For higher frequency phenomena we will use the original 30-seconds values. This means that we in the case of 10-minutes values are only interested in frequencies with periods from about 1.5 hour or longer (see 5.1 for a further discussion of this subject). A Butterworth filter with cut off period of $t_c = 50$ minutes will therefore be a good choice. Both the Butterworth filter with order 4 and the Butterworth filter with order 6 have steep enough amplitude response, and we choose the filter with lowest order.

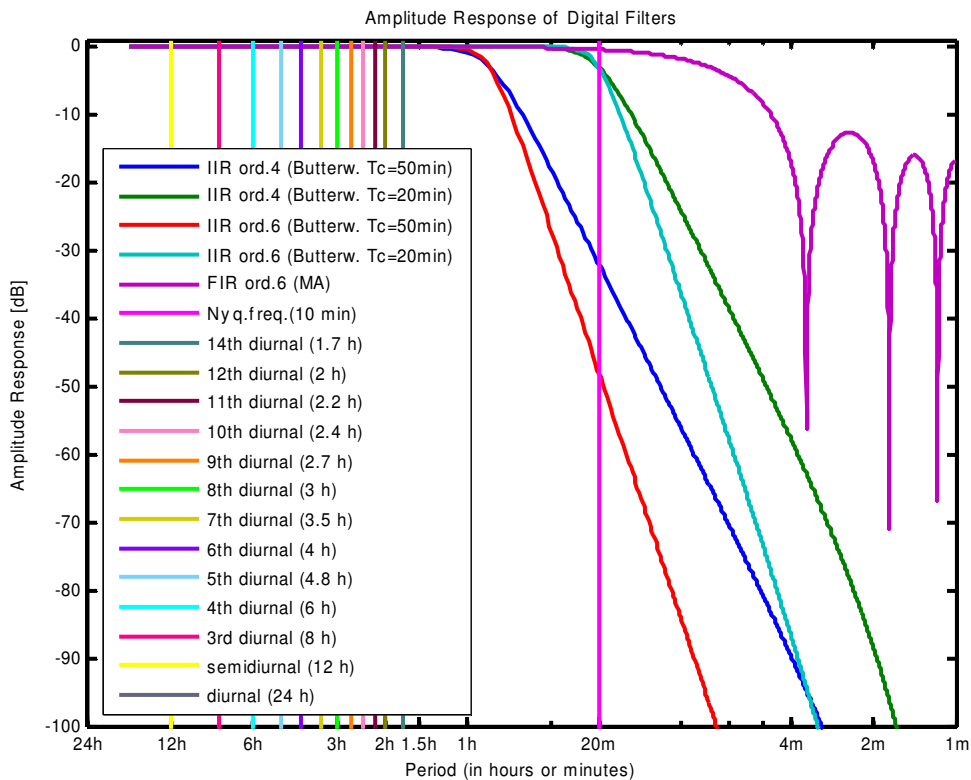


Figure 5-8 Amplitude response of 5 different filters (Butterworth and MA)

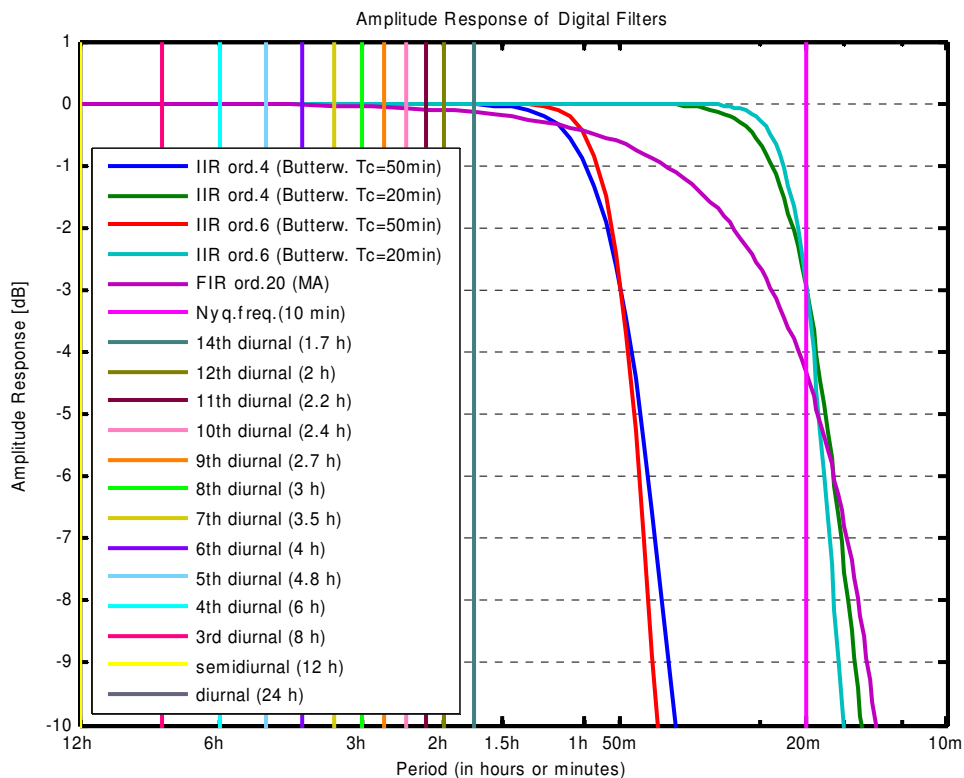


Figure 5-9 Amplitude response of 5 different filters (Butterworth and MA)

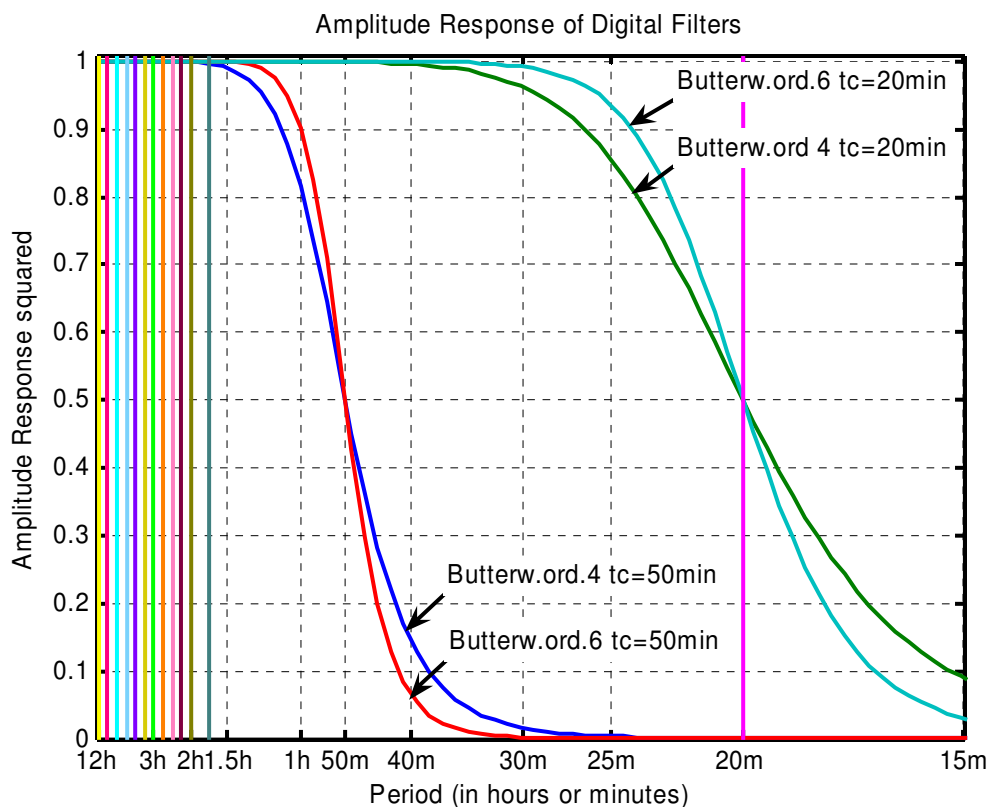


Figure 5-10 Squared amplitude responses of 4 Butterworth filters.

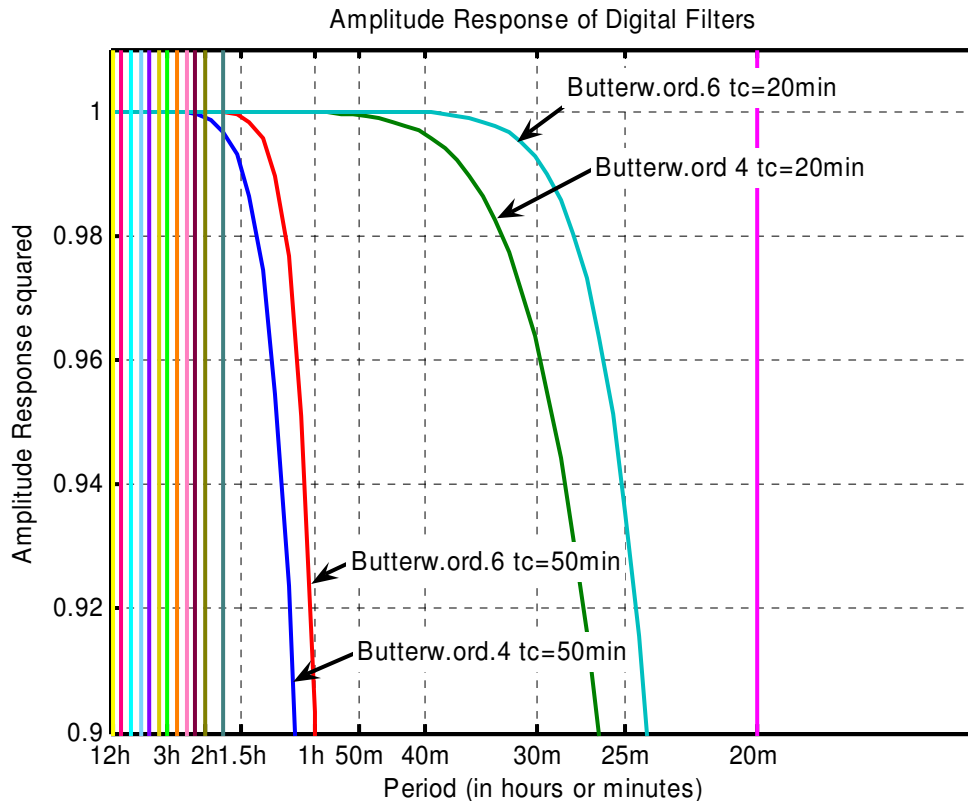


Figure 5-11 Squared amplitude responses of 4 Butterworth filters.

5.2 Filtering and decimating

In this section we test the NRT filters and DM filters using the data set from Stavanger (described in section 3 and section 4).

5.2.1 LP-filtering

The original and filtered data (two different filters) are shown together in Figure 5-12 (6 hours segment). We can observe that the signal, filtered with 4th order Butterworth filter ($t_c = 50$ minutes), is smoother than the signal filtered with the MA filter of order 20. (The corresponding decimated signals are shown in section 5.2.2 Figure 5-19.)

Other comparisons between data filtered with different filters are shown in Figure 5-13 - Figure 5-15.

The estimated PSD of the original and filtered data (different filters) are shown in Figure 5-16 - Figure 5-18. In Figure 5-16 we can observe that most of the signal with frequencies above the Nyquist frequency for 10-minutes sampling are attenuated both in the case of the 4th order Butterworth filter ($t_c = 50$ minutes) and the MA filter of order 20. However, the Butterworth filter attenuates best.

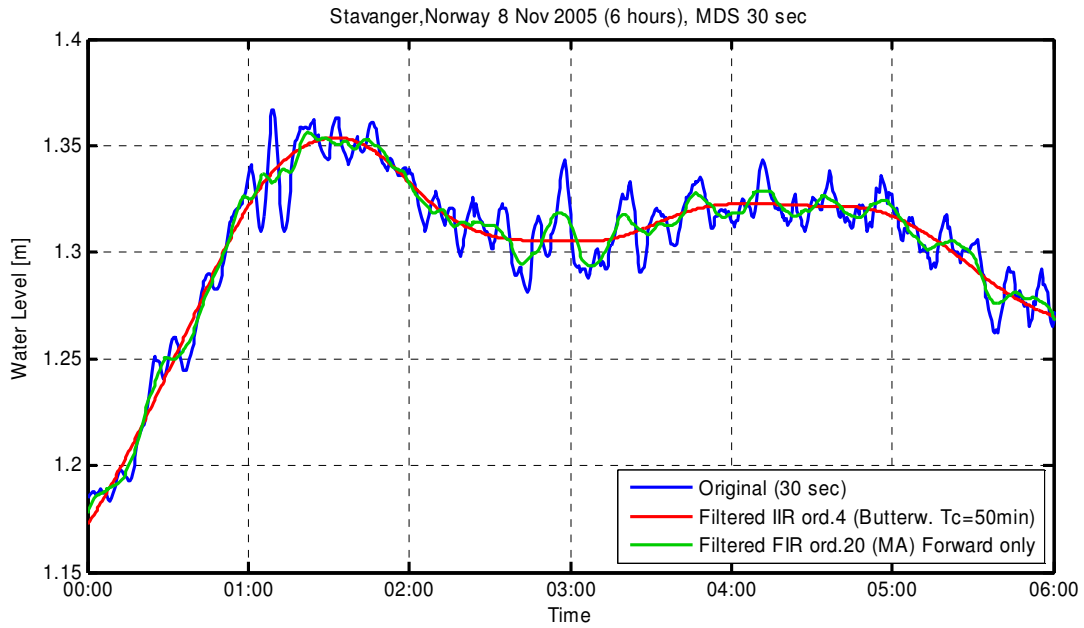


Figure 5-12 30-second data before and after filtering (two different filters).

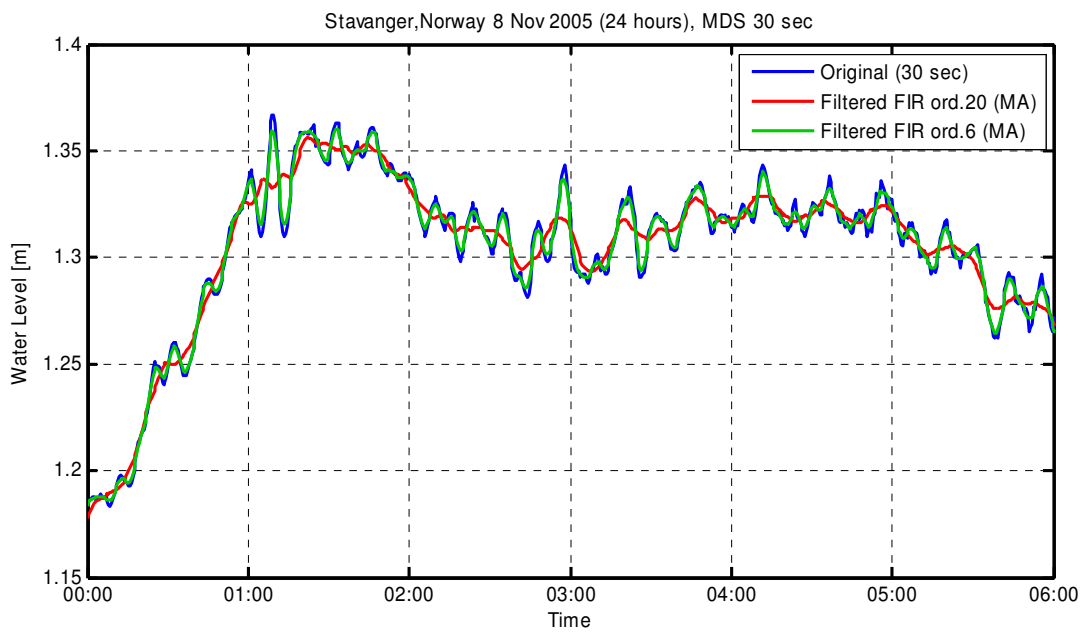


Figure 5-13 30-second data before and after filtering (two different filters).

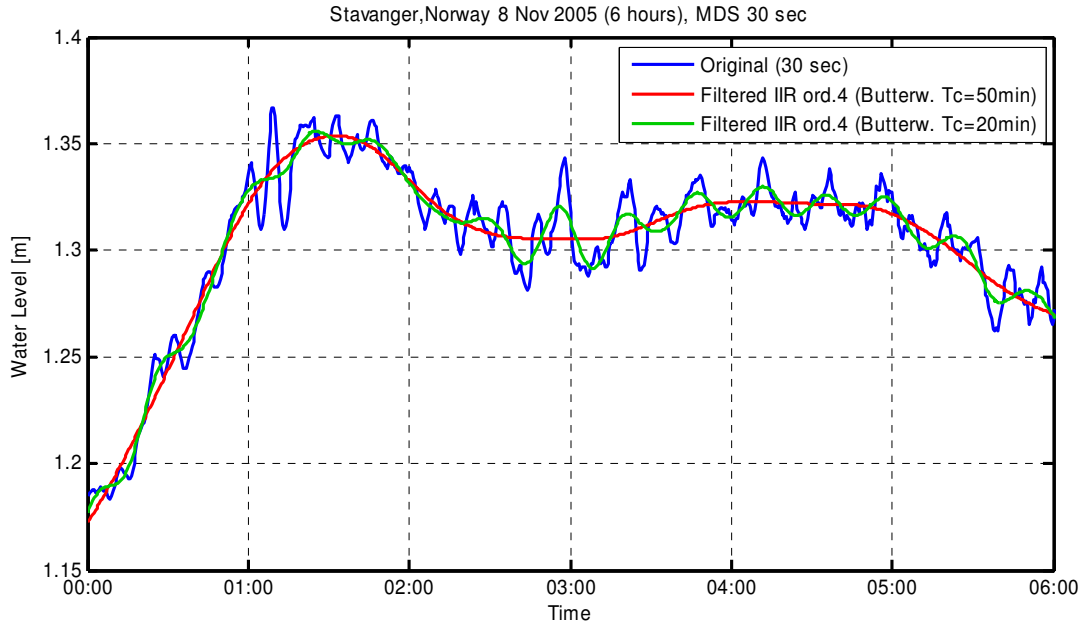


Figure 5-14 30-second data before and after filtering (two different filters).

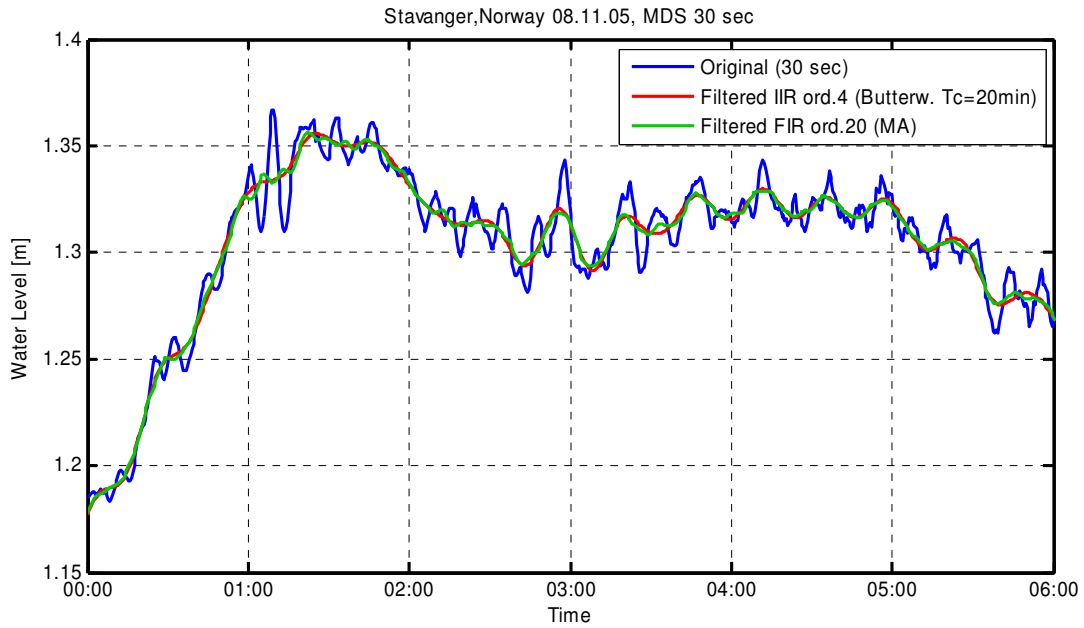


Figure 5-15 30-second data before and after filtering (two different filters).

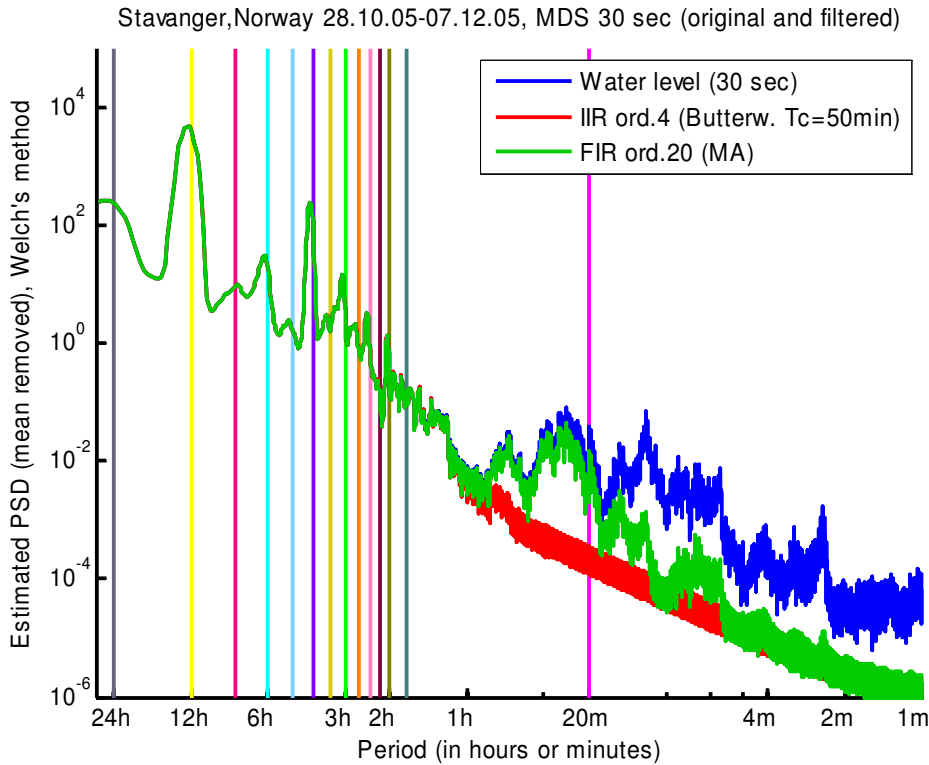


Figure 5-16 Estimated PSD of 30-seconds water level data (original and filtered with two different filters).

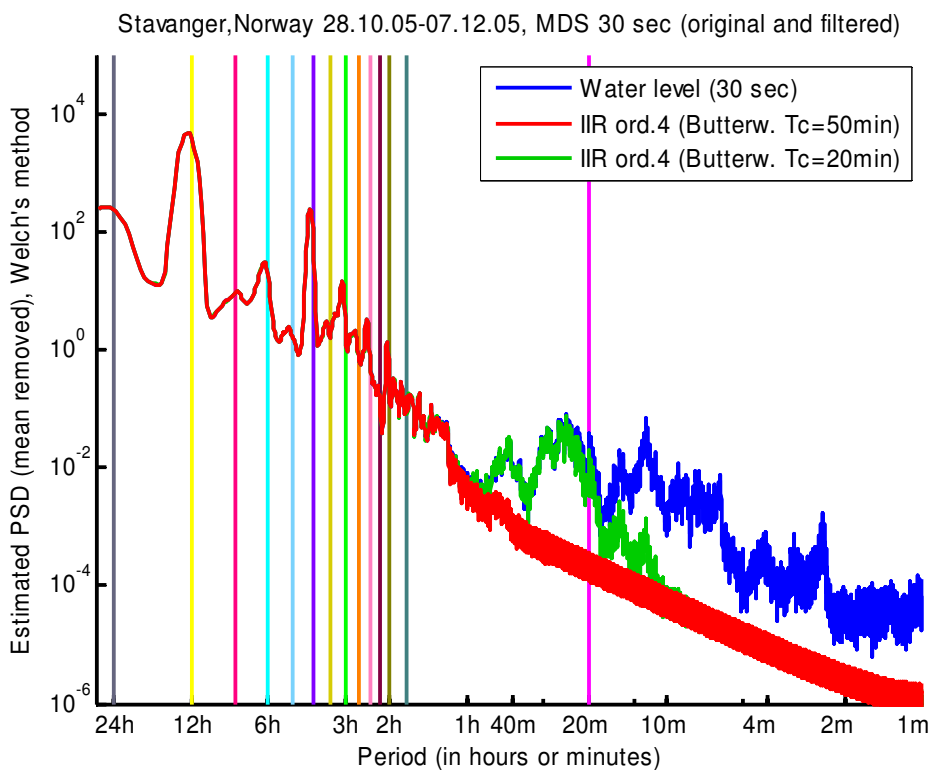


Figure 5-17 Estimated PSD of 30-seconds water level data (original and filtered with two different filters).

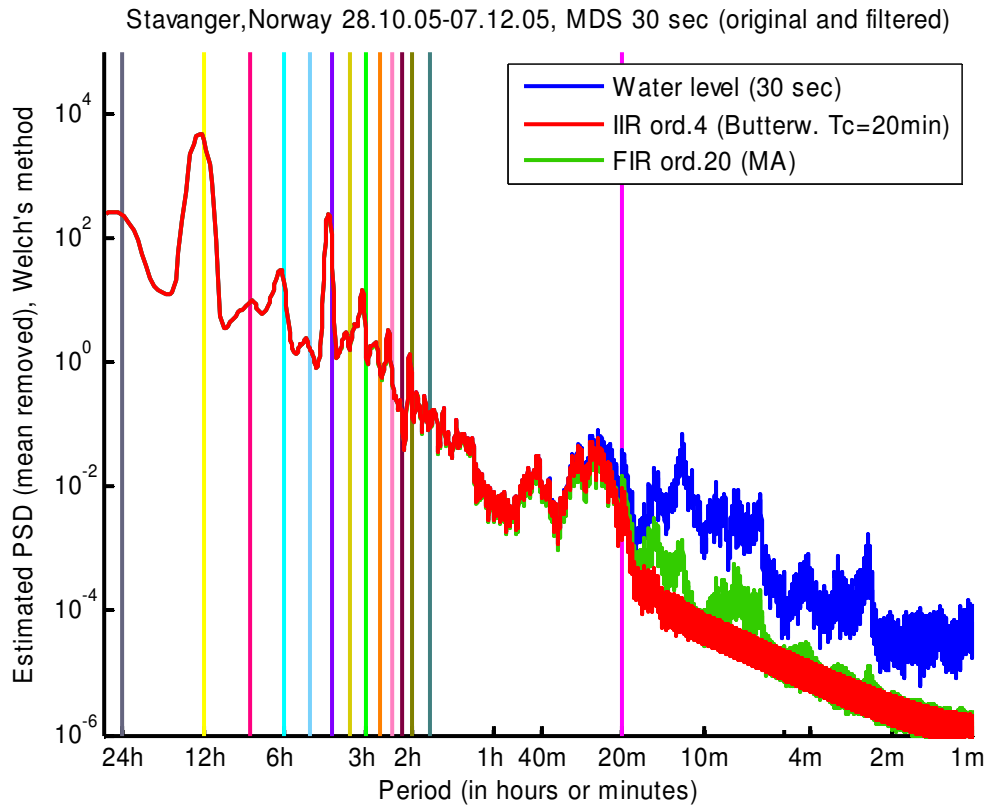


Figure 5-18 Estimated PSD of 30-seconds water level data (original and filtered with two different filters).

5.2.2 Decimating to 10-minutes values

After the data has been filtered it can be decimated to 10-minutes values as shown in Figure 5-19 - Figure 5-22 (different filters are used).

The difference between the 10-minutes values filtered with MA order 20 and Butterworth order 4 filter ($t_c = 50$ minutes) is shown in Figure 5-23 (24 hours) and Figure 5-24 (whole data segment). 95% of the differences are in the interval $[-0.0054\text{m}, 0.0053\text{m}]$. The largest difference is 0.015m.

The estimated power spectral density of the filtered and decimated time series is shown in Figure 5-25 - Figure 5-29.

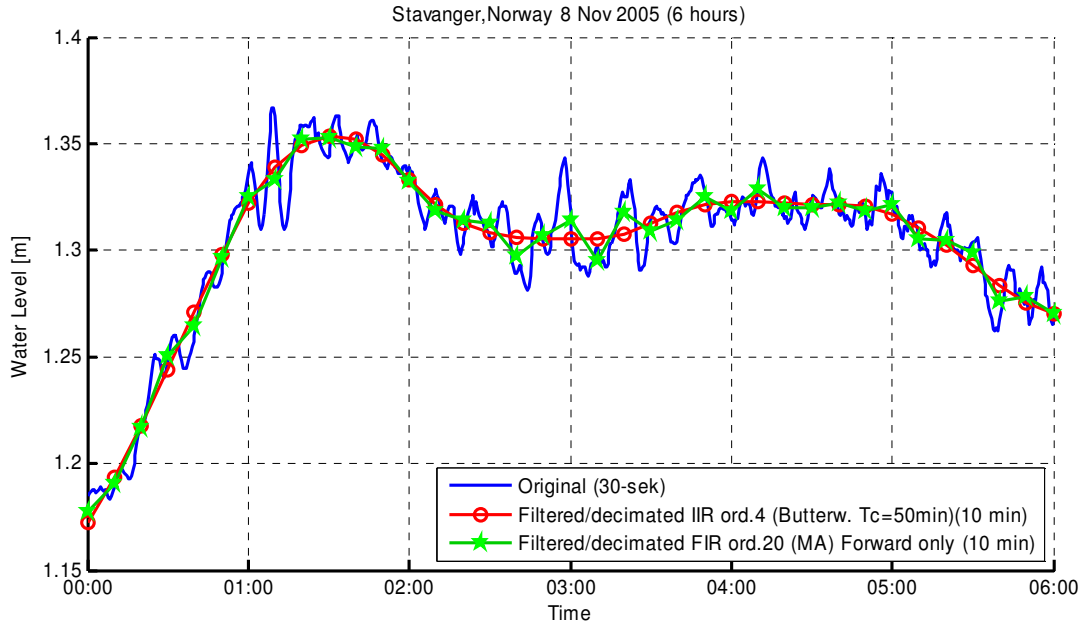


Figure 5-19 Water level: Original and filtered/decimated (two different filters).

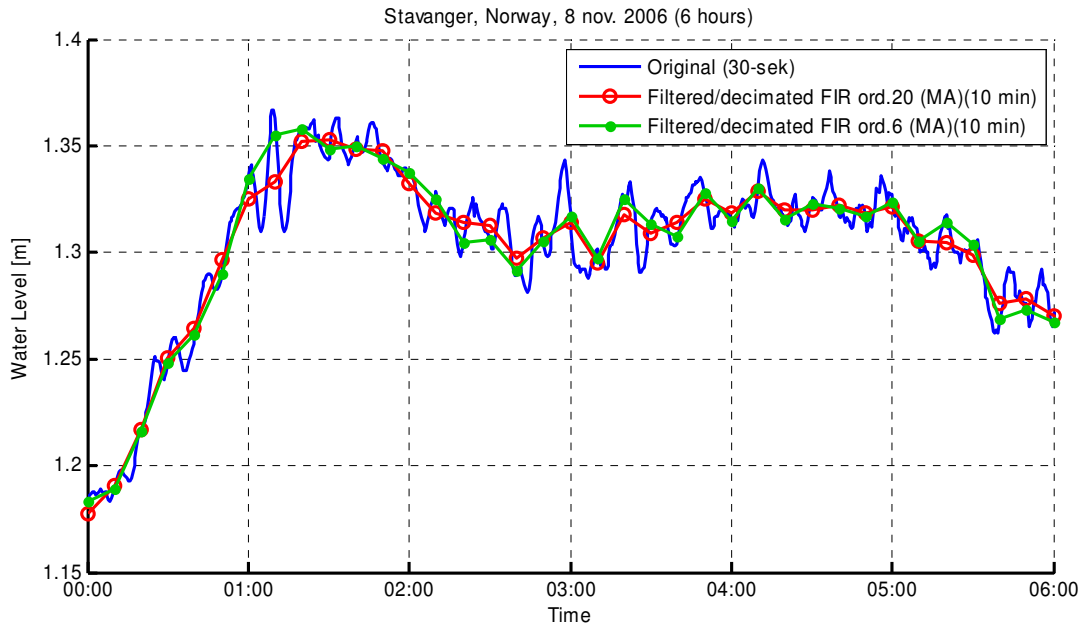


Figure 5-20 Water level: Original and filtered/decimated (two different filters).

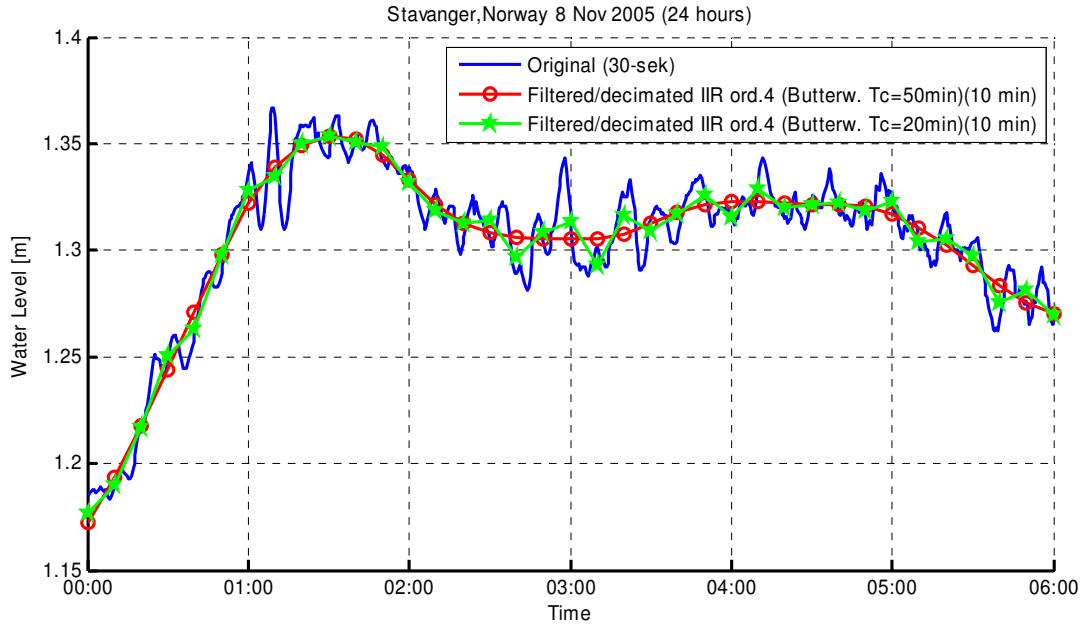


Figure 5-21 Water level: Original and filtered/decimated (two different filters).

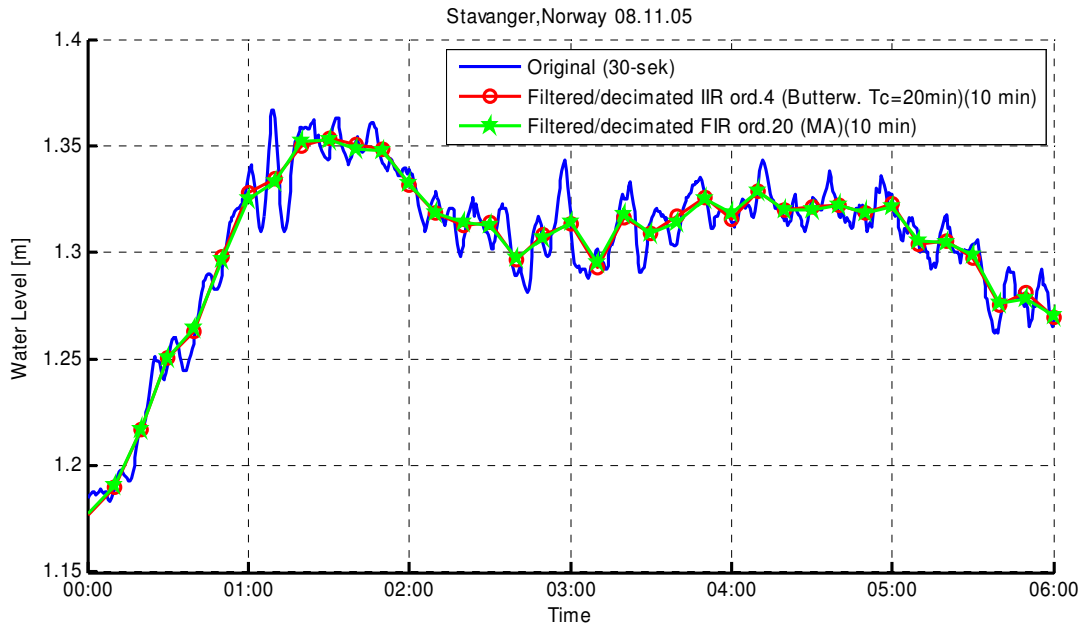


Figure 5-22 Water level: Original and filtered/decimated (two different filters).

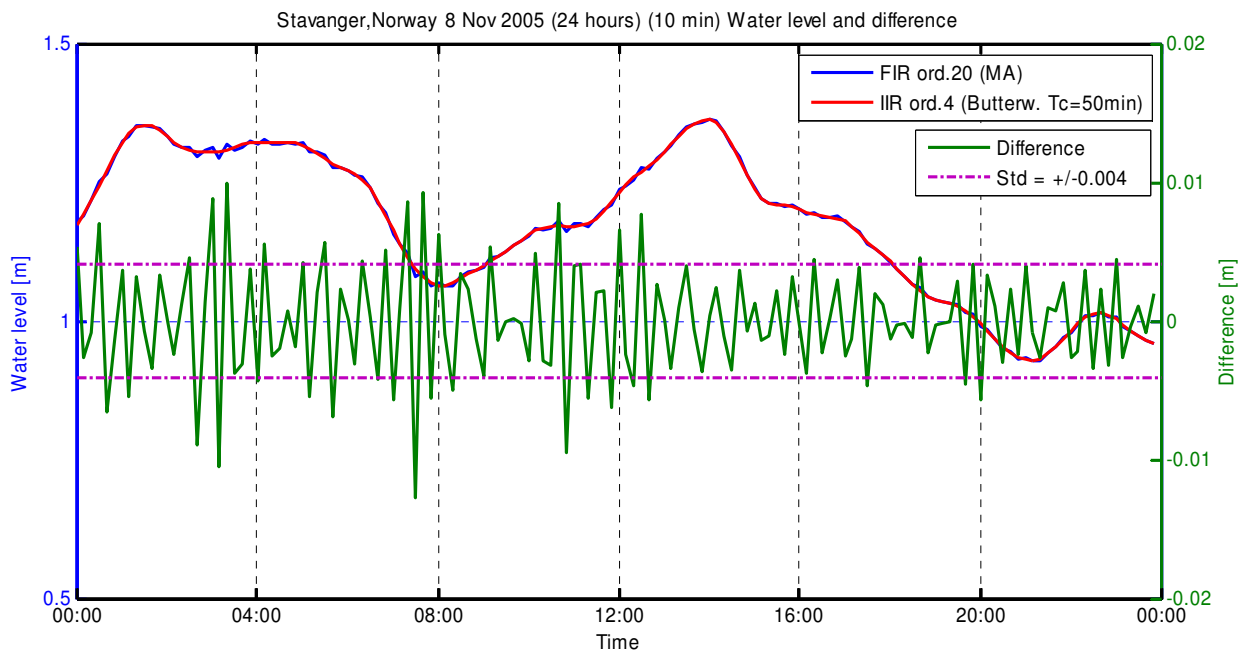


Figure 5-23 Water level filtered with 2 different filters and decimated (24 hours).

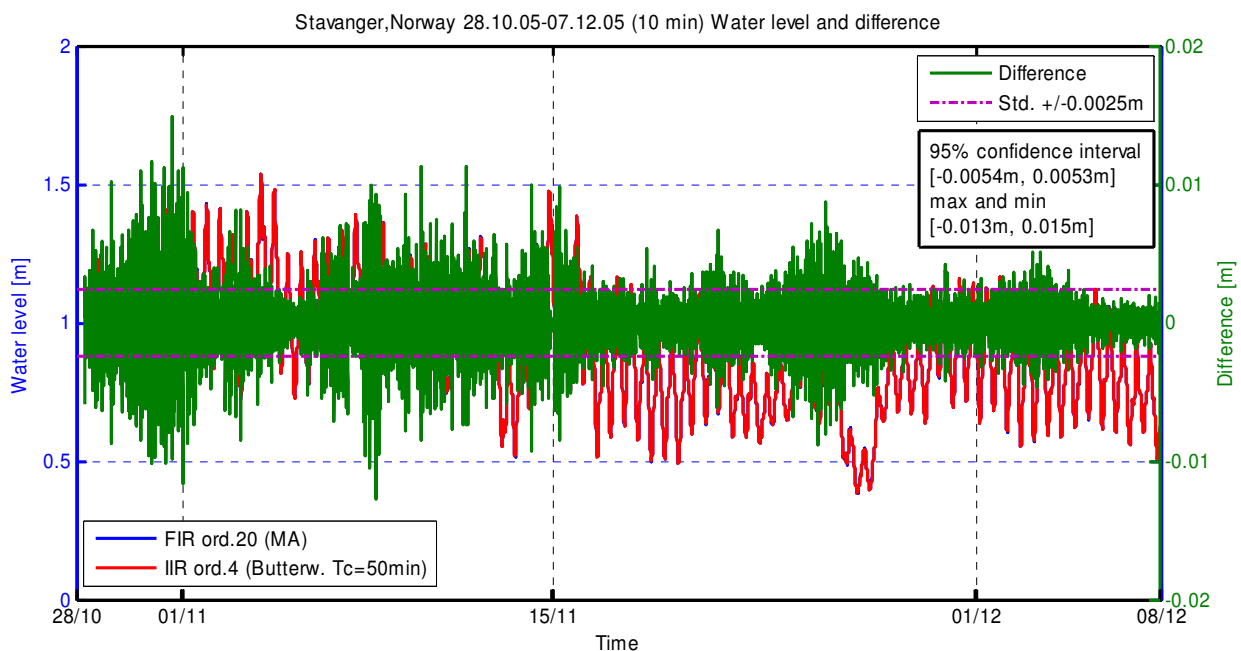


Figure 5-24 Water level filtered with 2 different filters and decimated.

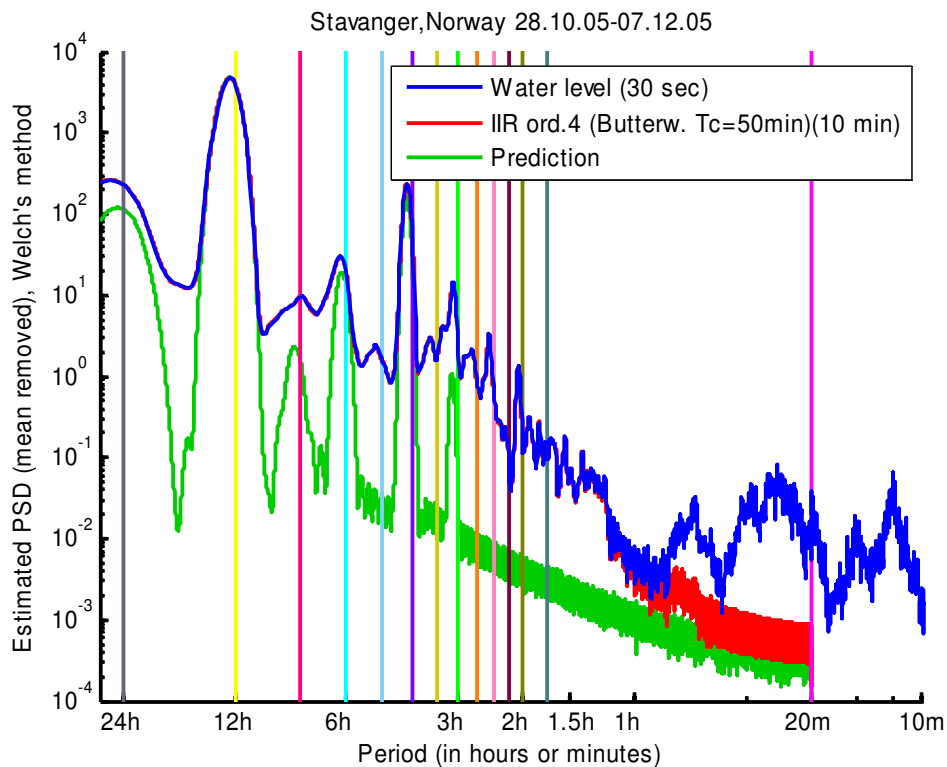


Figure 5-25 Estimated PSD of water level data (original, filtered/decimated and predicted).

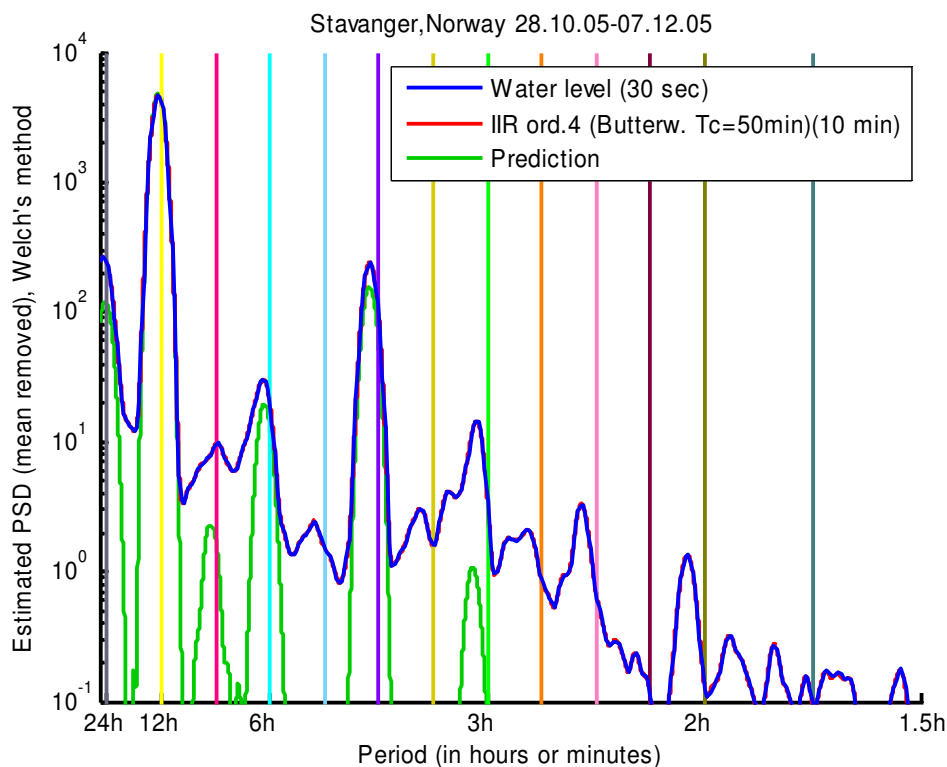


Figure 5-26 Same as Figure 5-25, focusing on tidal frequencies.

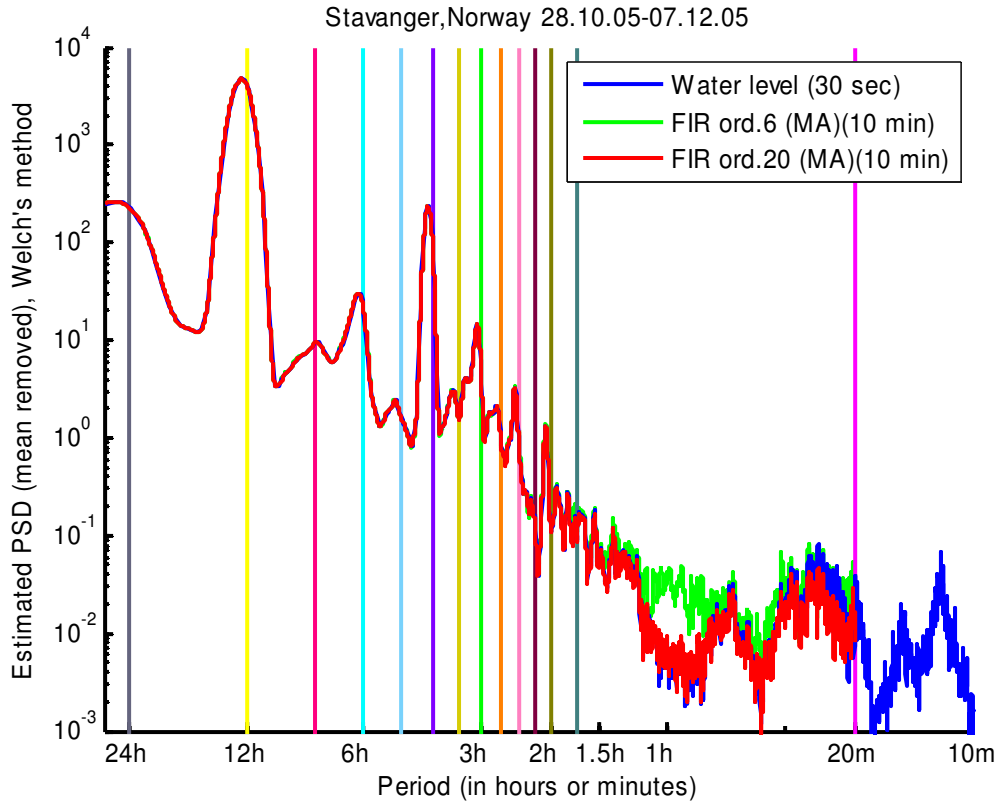


Figure 5-27 Estimated PSD of water level data (original and filtered/decimated).

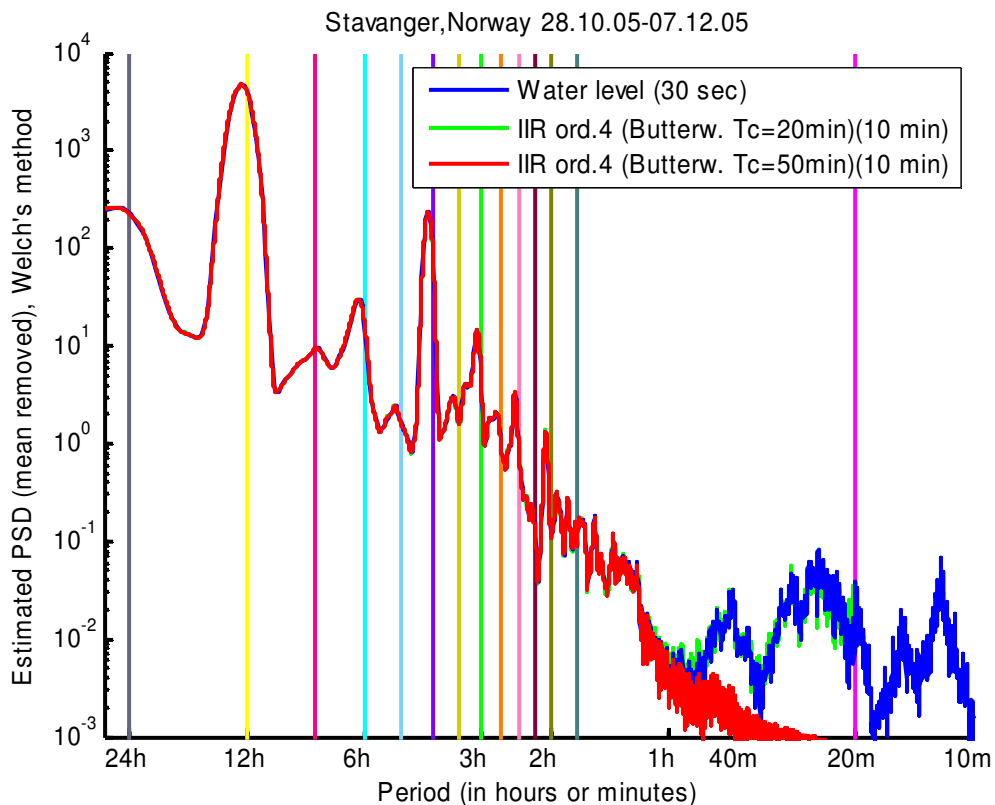


Figure 5-28 Estimated PSD of water level data (original and filtered/decimated).

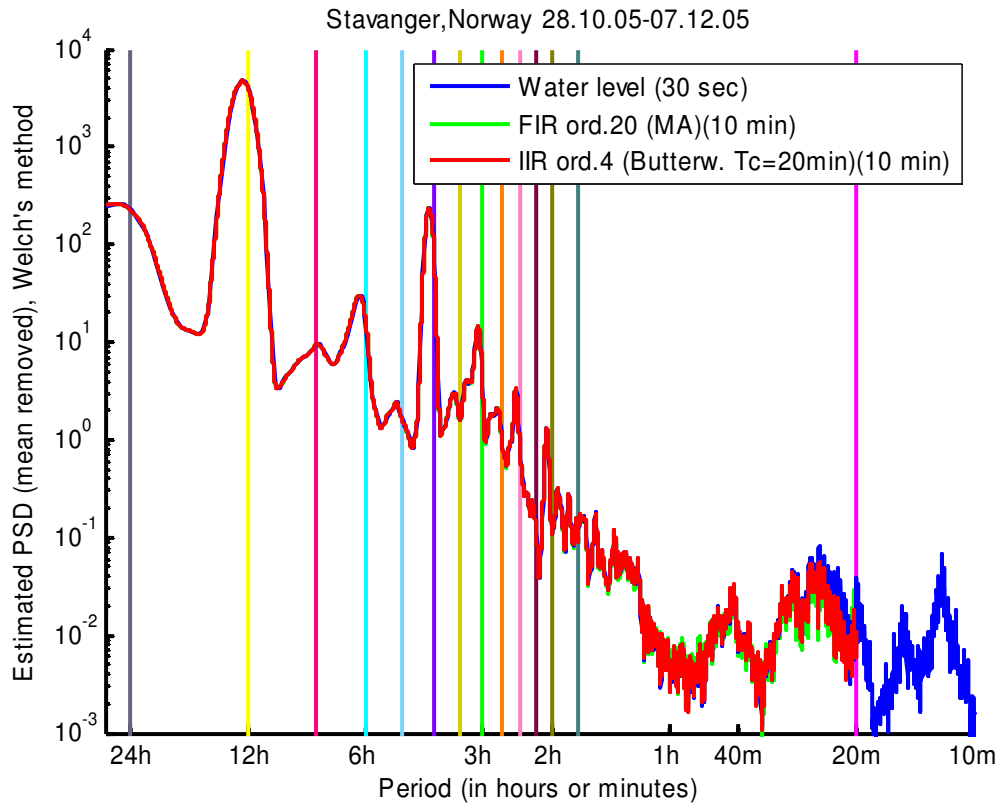


Figure 5-29 Estimated PSD of water level data (original and filtered/decimated).

6 Conclusions

6.1 30-seconds sampling

Sampling water level every 30-seconds will give a time series containing frequencies up to 1/60 Hz, which means periods down to 1 minute. The highest frequencies will never be reliable due to imperfect filtering in the datalogger.

30-seconds data must be quality controlled, but does not have to be filtered before distribution. However, decimated data should be filtered before decimating to avoid aliasing.

6.2 NRT anti-aliasing filter

To produce near real time 10-minutes values we recommend to filter the 30-seconds data with a moving average of order 20 (average over 10 minutes) before decimating to 10-minutes values. With a filter of order 20 the filter delay will be 5 minutes (anticipated to be the requirement, although not decided yet). The NRT 10-minutes values filtered with MA filter of order 20 will be suitable for analysing phenomena with period of about 1 hour or longer. The roll off is however very slow: For period 1 hour the amplitude response is 0.95 and for period 20 minutes it is 0.6. So there will be some energy between 1 hour and 20 minutes as well. The MA is not a good LP filter (slow roll off and bad stopband attenuation), but it is assessed to be satisfactory for the near real time data. It has a short delay, which is an advantage.

An even shorter moving average filter may be used for near real time 10-minutes values if the filter delay should be shorter than 5 minutes (remember filter delay requirement is not yet decided, but the requirements will hardly be more stringent than 5 minutes). However, these shorter filters actually have too high cutoff frequency to become an effective anti-aliasing filter.

If the delay can be longer than 5 minutes we could use a window-designed FIR filter for NRT data. It is shown in 5.1.1 that window-designed (Hamming) with order 60 and $t_c = 20$ minutes has a better amplitude response (lower stopband attenuation) than MA of order 20. However the delay is 15 minutes.

6.3 DM anti-aliasing filter

To produce delay mode 10-minutes values we recommend to filter the 30-seconds data with a Butterworth filter of order 4 and cutoff period of 50 minutes (filtered forward and backward) before decimating to 10-minutes values. The Butterworth filter is a good choice since it has a magnitude response that is maximally flat in the pass-band and monotonic overall. The roll off is fast as well. The DM 10-minutes data (filtered with this filter) will be suitable for analysing phenomena with period of about 1.5 hours or longer.

It is possible to choose a higher cut off frequency than $1/(50 \text{ minutes})$. The cutoff can be between $1/(50 \text{ minutes})$ and $1/(20 \text{ minutes})$. A higher cutoff frequency will lead to higher frequency components in the 10-min data series.

6.4 Comparing NRT and DM filter using test data from Stavanger

The filters have been tested with real data in section 5.2. We can see that in the case of the 42 days with data from Stavanger the difference of using the NRT filter (MA order 20) and the DM filter (Butterworth order 4, cutoff period 50 minutes) as anti-aliasing filter is small. 95% of the differences between the NRT 10-min values and the DM 10-min values are in the interval $[-0.0054\text{m}, 0.0053\text{m}]$. The largest difference is 0.015m. So we observe that although the amplitude responses of the two filters are quite different (the Butterworth has much steeper roll off and much better stopband attenuation than the MA) the effect on the filtered data is quite small. This result will of course depend on the data set used for the test. If the data set contain more of the higher frequencies (periods shorter than 1 hour) the difference will be larger.

7 References

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